1 2 3 4

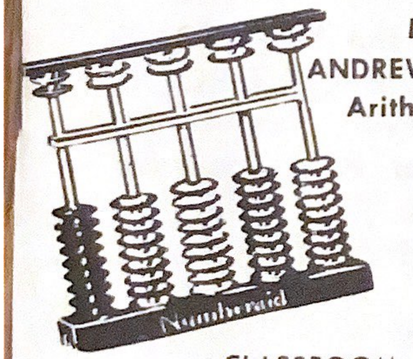
TRADE MARK

Numberaid

THE AMERICAN ABACUS

Guidebook
FOR PARENTS

Designed By
ANDREW F. SCHOTT, Ph. D
Arithmetic Consultant



CLASSROOM TESTED

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P. O. BOX 143 WAUWATOSA 13, WIS.

FOREWORD

The Numberaid Guidebook demonstrates how the Numberaid can be used in arithmetic. The instructions are written with the idea that the parents will work with the children. The arrangement of the guide has been carefully considered. It is written with the intent that each child should explore each chapter of the guidebook without .necessarily mastering it completely before going on to the next chapter. In other words, when the section on addition becomes too complex for a child, work in multiplication should be started. When multiplication becomes too complex, work in subtraction should be begun. In this way, the whole structure of arithmetic would be worked on as the children gain experience and maturity. Needless to say, the rate of various individuals' comprehension and their ability to complete the work outlined will vary greatly. This should not disturb the parents. The important point is that the opportunity of learning something about every phase of arithmetic should be available to every child at his level of understanding. Games which promote understandings, develop skill, and motivate retention are included in the guidebook. Work with number becomes a pleasant, fun-filled experience when the Numberaid is used as a teaching-learning tool.

There are two editions of this NUMBERAID GUIDE, the NUMBERAID GUIDE FOR PARENTS and the NUMBERAID GUIDE FOR CLASSROOM use. The NUMBERAID GUIDE FOR PARENTS comes with every Numberaid. The classroom guide can be obtained by special order from *Tools for Education Inc.*, P. O. Box 143, Wauwatosa 13, Wisconsin.

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Counting

Quantity and Process

Addition

Multiplication

Subtraction

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Getting Acquainted with Your Numberaid. Put the Numberaid flat on a table or desk, with the green counters on the right. Be sure the trademark "Numberaid" is at the bottom. Figure I shows the various parts of the Numberaid, such as the stop, the counters, the rods, the top and bottom bars.

Figure 1 Numberaid Cleared
(No counters at stop)

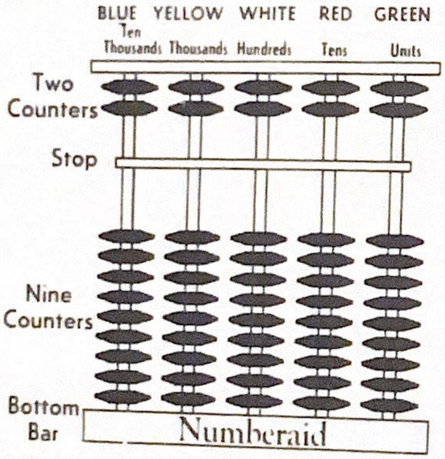
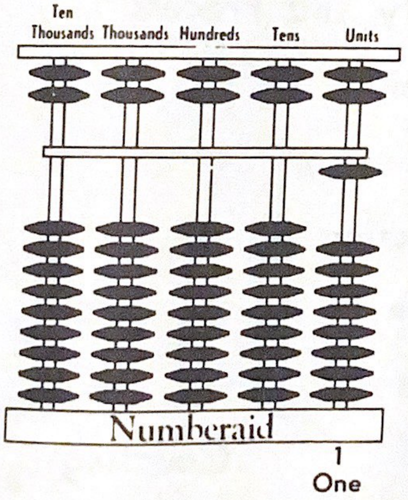


Figure 2 The First



Clear the Numberaid. Move all the counters as far as they will go away from the stop (Figure I). When that is done, you have "Cleared your Numberaid." It looks exactly like Figure I. When the Numberaid is Cleared, nothing is shown on it. It has no numbers on it. Zero is shown on each rod.

Count to One. Clear your Numberaid. Be sure all the counters are as far as they will go away from the stop. Move one green counter up to the stop (Figure 2). Say "One." Move the counter back from the stop. Say "Zero." The rod with the green counters is called the units rod. One is a unit. Zero is only a place holder. Practice counting from zero to one and back to zero. Play Game I.

Figure 3 The Second

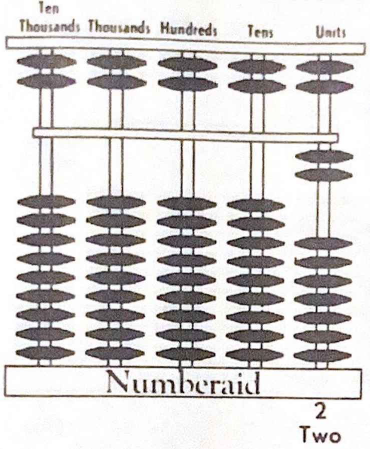
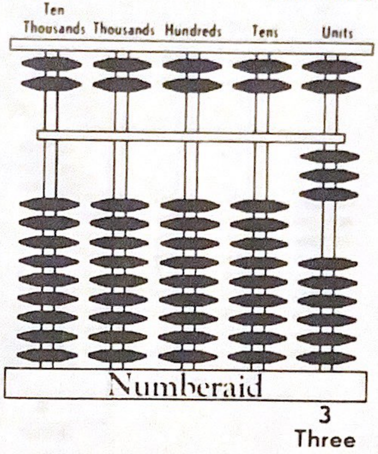


Figure 4 The Third



Count to Two. Begin by clearing your Numberaid. Say "Zero." Count to one (1) and move the first green counter up to the stop. Say "One." To count to two (2), move the second green counter up until it touches the first. Say "Two." Look at Figure 3 to be sure you are right. Count back to one, as you push down the second counter. Say "One." Count back to zero as you push down the first counter. Say "Zero." Practice counting from zero to two and back to zero.

Count to Three. To count to three (3), begin by clearing your Numberaid, saying "Zero." Count "One." Count "Two." To count to three, move the third green unit-counter up until it touches the second. Say "Three." Look at Figure 4 to be sure you are right so far. Count back to two, one, then to zero. Then play Game 2.

Figure 5

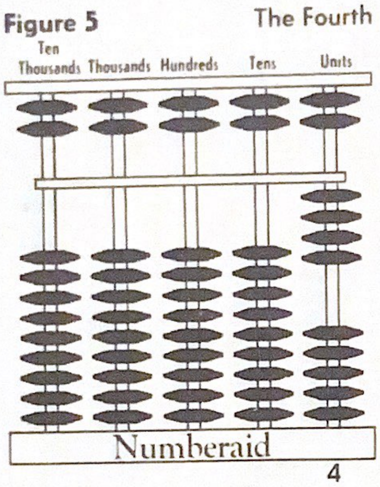
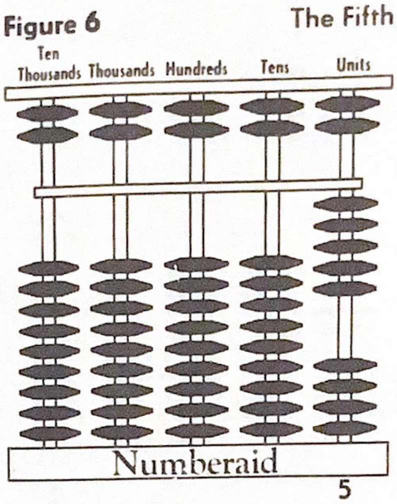


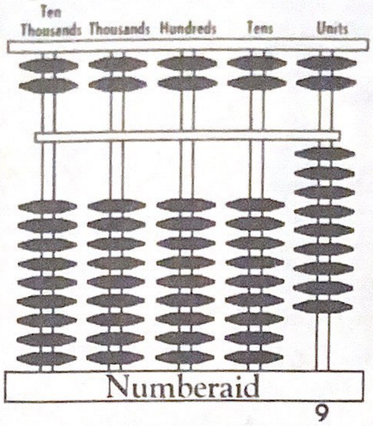
Figure 6



Count to Four. To count to four (4), begin by clearing your Numberaid. Say "Zero." Count to three as you did before, saying the numbers as you go. To count to four, move the fourth green unit-counter up until it touches the third. Say "Four." Look at Figure 5 to be sure you are right so far. Count back to zero by pushing the counters down one by one, as you have done before, counting aloud as you go. Now play Games 3 and 4.

Count from Five to Nine. To count to five (5), begin by clearing your Numberaid, saying "Zero." Count to four as you did before, saying the numbers as you go. Then count to five, and move the fifth green unit-counter up until it touches the fourth. Say "Five." Look at Figure 6 to be sure you are right so far. Now play Game 5.

Figure 7 The Ninth



To count to six (6), move the sixth green unit-counter up until it touches the fifth. Say "Six." To count to seven (7), move the seventh green unit-counter up until it touches the sixth. Say "Seven." To count to eight (8), move the eighth green unit-counter up until it touches the seventh. Say "Eight." To count to nine (9), move the ninth green unit-counter up until it touches the eighth. Say "Nine." (See Figure 7.) After you have counted to nine, count back to eight by pushing the ninth counter down as far as it will go. Say "Eight." Count back to seven, to six, to five, to four, to three, to two and to one, then to zero.

Count various similar objects, one kind at a time, which are available, such as chairs, stairs, windows, etc. In each case, be sure to return to zero. Play Game 6.

Figure 8 The Tenth

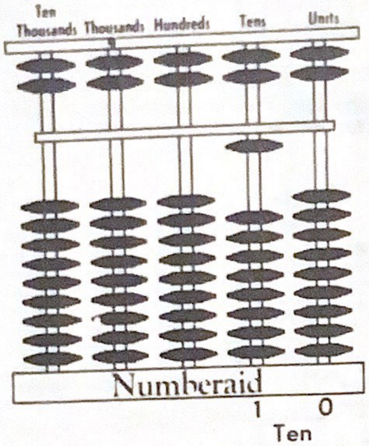
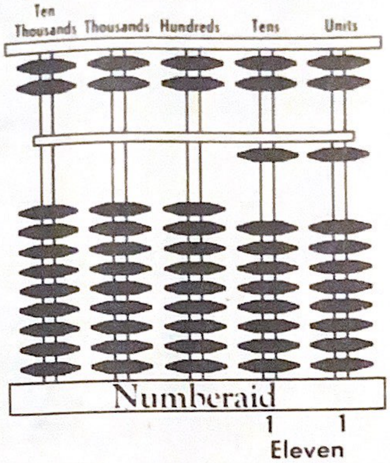


Figure 9 The Eleventh



Count to Ten. You have already learned to count to nine. To count to ten (10), first count to nine. Then move down all of the green counters that you used to count to nine (9), and move up one red counter on the rod next to the green ones. As you do this, say "Down go the nine green counters and up goes one ten-counter." There must be one red ten-counter and no (zero) (0) green unit-counters at the stop to show 10. Ten (10) on the Numberaid looks like Figure 8. Count back to nine, saying, "Down goes the ten, up comes the nine," while you push the ten-counter down and the nine one-counters up to the stop. Then you can count back to zero easily. Play Games 7 and 8.

Count to Eleven. First count to 10. To count to eleven, push up a green unit-counter to the stop, while you say "Eleven." Eleven (11) is shown on your Numberaid by one red ten-counter and one green unit-counter next to the red ten-counter (Figure 9). Then to count back to zero, push the green unit-counter down first and say, "Ten"; then, "Down goes ten and up comes nine." As each remaining unit-counter is pushed down, say "Eight, Seven, Six," etc., until the Numberaid is cleared.

Figure 10 The Twelfth

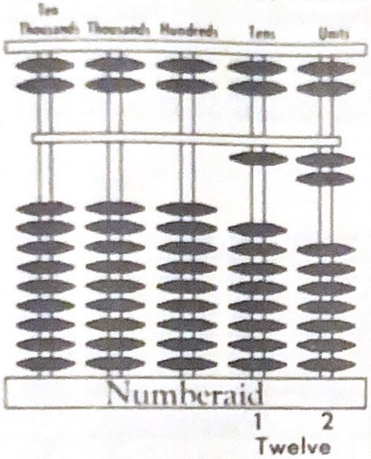


Figure 11 The Thirteenth

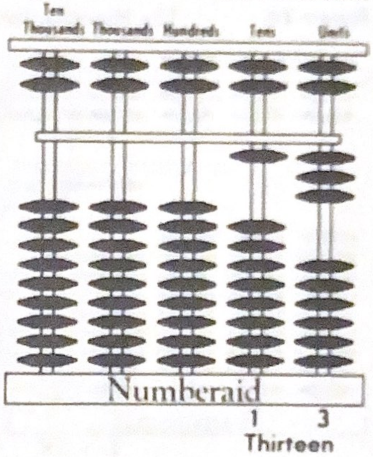


Figure 12 The Fourteenth

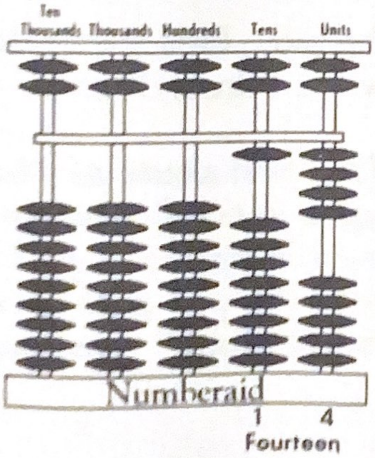
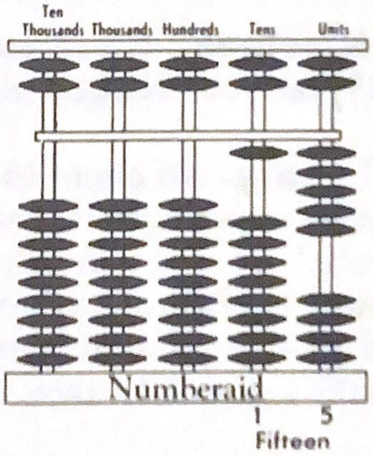


Figure 13 The Fifteenth



Count to 12 on your Numberaid (Figure 10); and then to 13 (Figure 11); to 14 (Figure 12); to 15 (Figure 13) and then continue to 18. Plav Game 9.

Figure 14 The Nineteenth

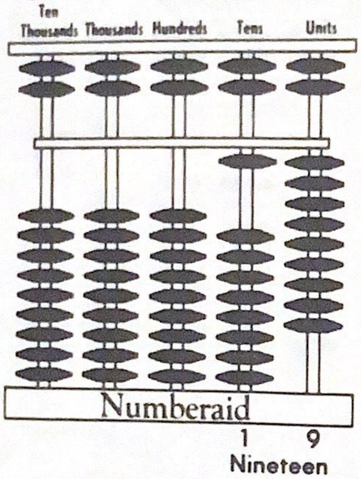
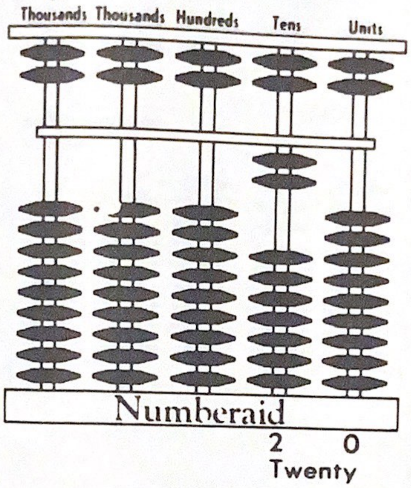


Figure 15 The Twentieth

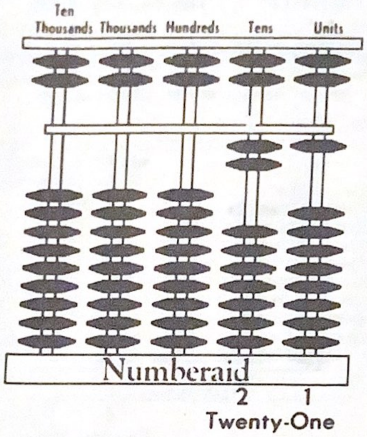


Count to Nineteen. Clear your Numberaid. First count to eighteen. Nineteen (19) on your Numberaid is shown in Figure 14.

Count to Twenty. To count to twenty (20), first count to 19. Then push down the nine one-counters and push up a second ten-counter as you count "Twenty." Twenty on your Numberaid looks like Figure 15.

Then count backward, saying, "Nineteen (as you push down one ten-counter and push nine unit-counters up to the stop)." Continue counting backward until you get to zero.

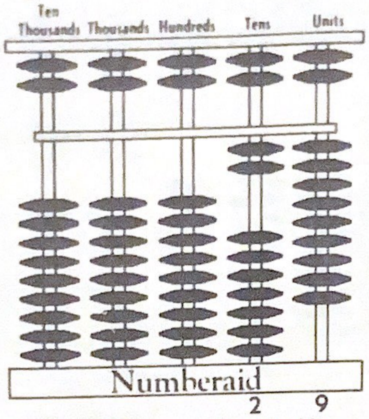
Figure 16 The Twenty-First



Count to Twenty-one. To count to twenty-one (21), first count to 20. Twenty-one on the Numberaid is shown in Figure 16. Then count backward to zero.

Count various similar objects nine to 21, both forward and backward. Count the number of times a ball is bounced, cars passing on the street, and other things of a like nature. Then play Game 10.

Figure 17 The Twenty-Ninth

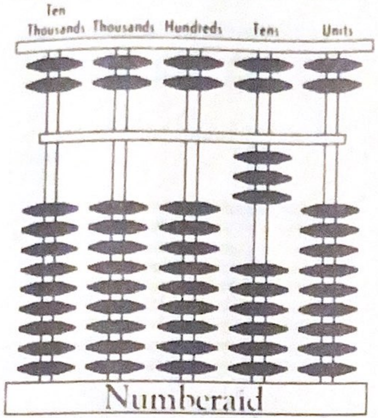


Twenty-Nine

Count to One Hundred. Now you can count to one hundred (100). Here are some drawings to check as you count.

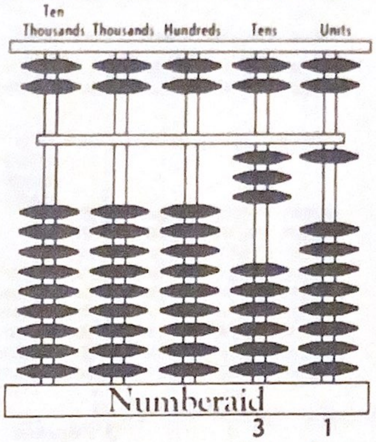
Twenty-nine	(29)	Figure 17
Thirty	(30)	Figure 18
Thirty-one	(31)	Figure 19
Ninety-nine	(99)	Figure 20
One Hundred	(100)	Figure 21

Figure 18 The Thirtieth



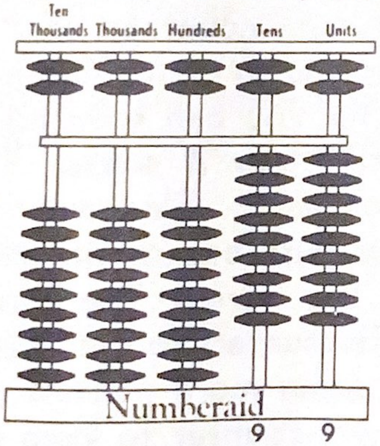
3 0
Thirty

Figure 19 The Thirty-First



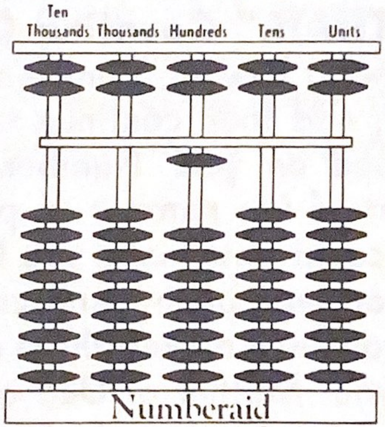
3 1
Thirty-One

Figure 20 The Ninety-Ninth



9 9
Ninety-Nine

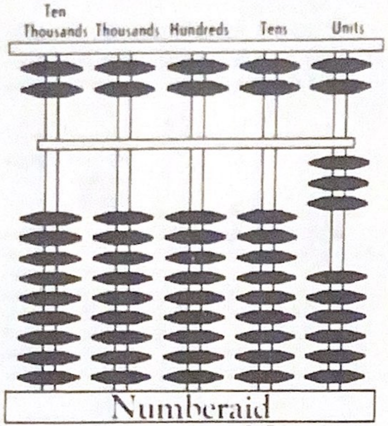
Figure 21 The Hundredth



1 0 0
One Hundred

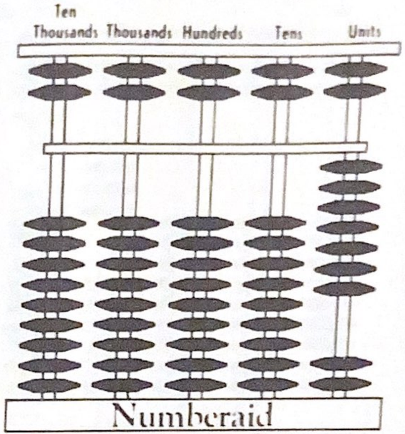
Count various similar objects (pennies, marbles, beans, etc.) which range in number from 10 to 100, keeping count on your Numberaid. Play Game 6.

Figure 22 Counting Pennies



3 Pennies
Three Pennies

Figure 23 Counting Pennies



7 Pennies
Seven Pennies

"MORE THAN" and "LESS THAN"

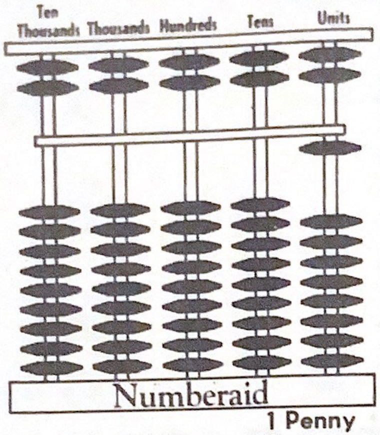
"MORE THAN," Counting Ones. You can count on your Numberaid now. Knowing how to count makes it possible to count to a number, stop for awhile, and then continue to count; for you can keep track of where you stopped on your Numberaid. Here's how it works. You want to keep count of the number of pennies you have. Clear your Numberaid. Use it to count how many you have. You push one green unit-counter to the stop for each penny, and count one, two, three. You have three pennies and you have moved three green unit-counters to the stop (Figure 22).

Then your Mother or Dad gives you some more pennies. As he gives you each penny, you push up a green unit-counter to keep count. One, and another, and another, and another. You have kept count on your Numberaid. You can count the number of green unit-counters you have moved up to the stop: one, two, three, four, five, six, seven. You can say you now have seven (7) pennies (Figure 23). These are more pennies than there were at first.

If you write these numbers on paper as soon as you show them on your Numberaid, you have a record which you can keep to show the work you have done. This is very important.

A number of similar objects (such as beans, etc.) should be shown to the children and counted by them on their Numberaids. Then an additional number of the same kind of objects should be shown to the children, while they continue to count to find out how many were shown in all.

Figure 24 Counting Pennies



1 Penny
One Penny

Use different situations, such as bouncing a ball, having a child skip first on the left foot, then on the right. Endless variations are possible. As the children gain understanding of the process and skill in counting ones to nine, the following problems can be used. Be sure they are represented by concrete objects.

The first problem could be stated as follows: "Here is a penny. How many pennies do I have? I will count no more, or zero more pennies. How many pennies did I count in all?"

Your Numberaid should look like Figure 24.

After many concrete objects have been used and real experiences with counting have been gained, imaginary objects can be used to take their place. Imagination is thereby developed. Play Game 6.

1 1 1 1 1 9 9 9 9 9
0 2 4 6 8 1 3 5 7 9

2 2 2 2 2 10 10 10 10
0 1 3 5 7 2 4 6 8

3 3 3 3 11 11 11 11
0 2 4 6 1 3 5 7

4 4 4 12 12 12
1 3 5 2 4 6

5 5 5 13 13 13
0 2 4 1 3 5

6 6 14 14
1 2 2 4

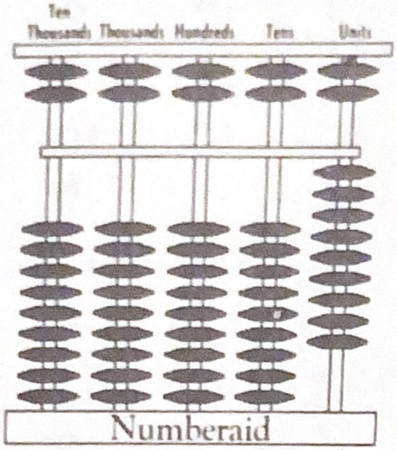
7 7 15 15
0 2 1 3

8 16
1 2

9 17
0 1

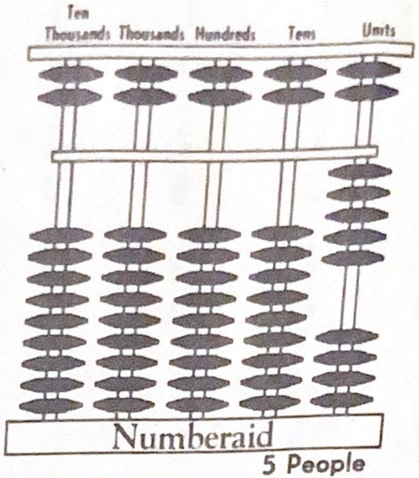
0 0 0 0 0 18
0 2 4 6 8 1

Figure 25 Counting People



9 People
Nine People

Figure 26 Counting People



5 People
Five People

"Less Than," Counting Ones. Imagine there are nine people in a room. You show this by moving nine green unit-counters up to the stop on your Numberaid (Figure 25). One person leaves the room, then another one, another one, and another one. As each one leaves you count backward from nine, one green counter at a time. When all those who wanted to go have gone, you stop counting backward. You count the number of green counters left against the stop (Figure 26). There are five. You know there are five people left in the room, without having to count them again. There are less than the number of people present in the beginning.

The following problems can be used as the basis for practice. The first subtraction problem can be stated as follows: "If I have no pennies and I do not take any away, how many will there be?"

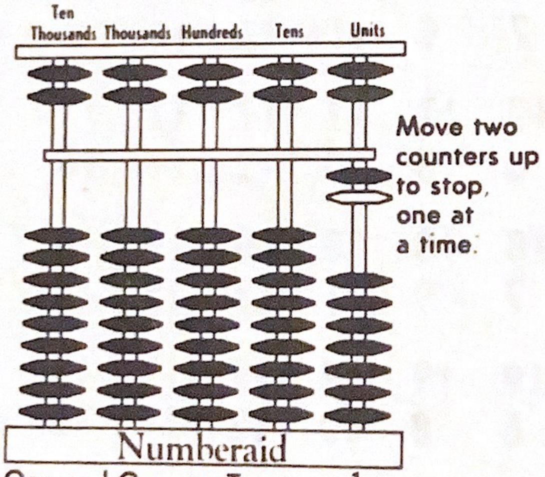
0	2	4	6	8	10	10	10	10	10					
0	0	0	0	0	1	3	5	7	9					
1	3	5	7	9	11	11	11	11	11	11				
1	1	1	1	1	0	2	4	6	8	10				
0	3	5	7	9	12	12	12	12	12	12				
2	2	2	2	2	1	3	5	7	9	11				
	3	5	7	9	13	13	13	13	13	13	13			
	3	3	3	3	0	2	4	6	8	10	12			
0		5	7	9	14	14	14	14	14	14	14			
4		4	4	4	1	3	5	7	9	11	13			
			6	8	15	15	15	15	15	15	15	15		
			5	5	0	2	4	6	8	10	12	14		
0			7	9	16	16	16	16	16	16	16	16		
6			6	6	1	3	5	7	9	11	13	15		
0			8		17	17	17	17	17	17	17	17	17	
7			7		0	2	4	6	8	10	12	14	16	
			8	9	18	18	18	18	18	18	18	18	18	
			8	8	1	3	5	7	9	11	13	15	17	
0				9	19	19	19	19	19	19	19	19	19	19
9				9	0	2	4	6	8	10	12	14	16	19

QUANTITY AND PROCESS

One. Count from zero to one, and back to zero, on your Numberaid. Count from zero to one, to two, to three, to four, and then to five, o your Numberaid. One (shown on the Numberaid by one green unit counter) is the amount of difference between consecutive numbers such as 1, 2, 3, etc. One is the building block we use to build numbers. All ones are equal, so we can say that any green unit-counter is the same or is equal to any other green unit-counter. This is an important new way to work with numbers, and is different from their use in counting.

Two. Build two by adding ones (Figure 27). There is a quicker way to show two on your Numberaid. You can take two counters at a time (Figure 28). If you build two by adding ones, you can see that two is made up of two ones, and that the ones are the same, or equal to one another (Figure 27).

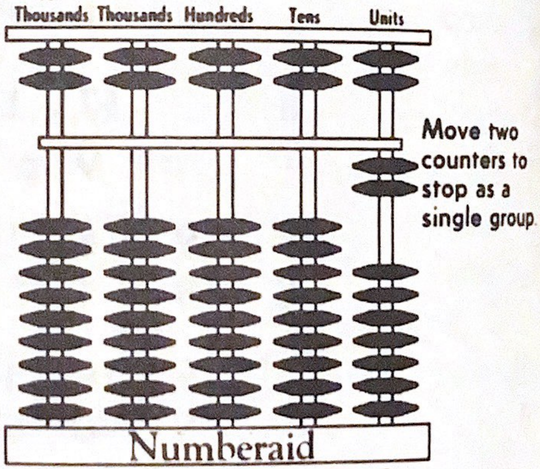
Figure 27 1+1=2



Move two counters up to stop, one at a time.

One and One are Two $\begin{array}{r} 1 \\ +1 \\ \hline 2 \end{array}$

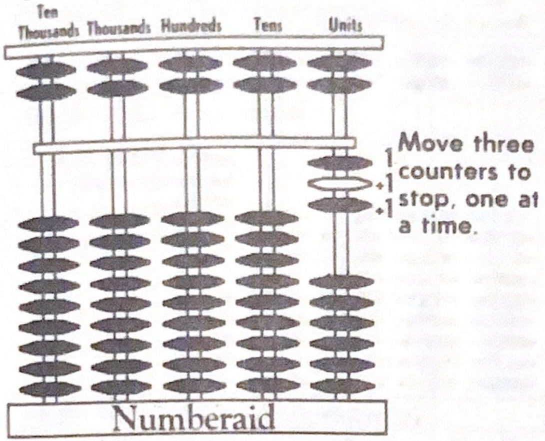
Figure 28 Two (2)



Move two counters to stop as a single group.

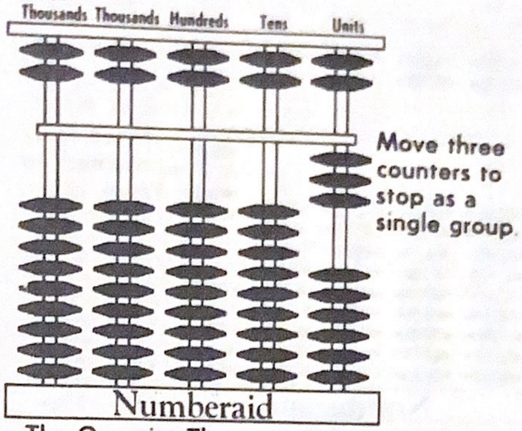
The Quantity Two 2

Figure 29 1+1+1=3



One and One and One are Three 3

Figure 30 Three (3)

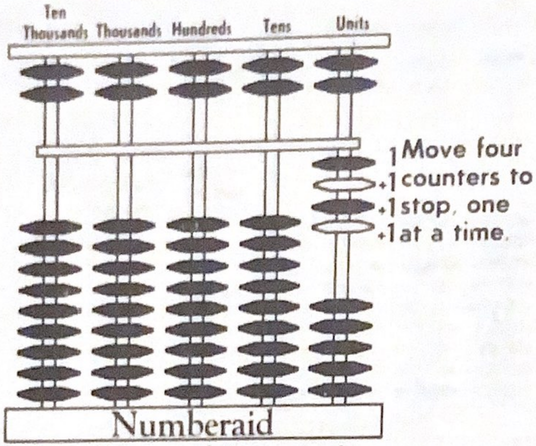


The Quantity Three 3

When you show two, you do not break it up into ones, but move two unit-counters directly to the stop. When you say "Two trees," or "Two apples," or "Two anything," you do not have to count, "one, two," but you can show this number by pushing two green unit-counters up to the stop. This is using number in a brand new way.

Three. Build three by ones, one green unit-counter at a time (Figure 29) on your Numberaid. Then put the number three on your Numberaid, taking three green unit-counters at a time (Figure 30).

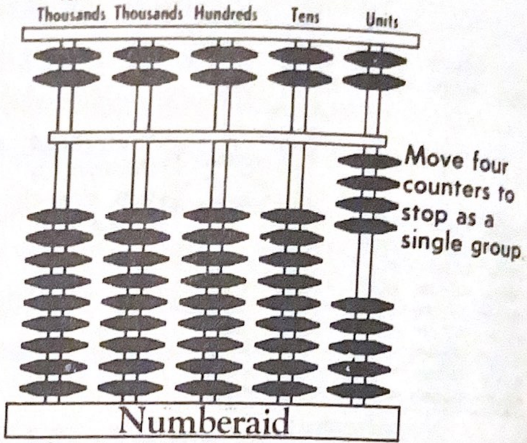
Figure 31 1+1+1+1=4



Move four counters to stop, one at a time.

One and One and One and One are Four 4

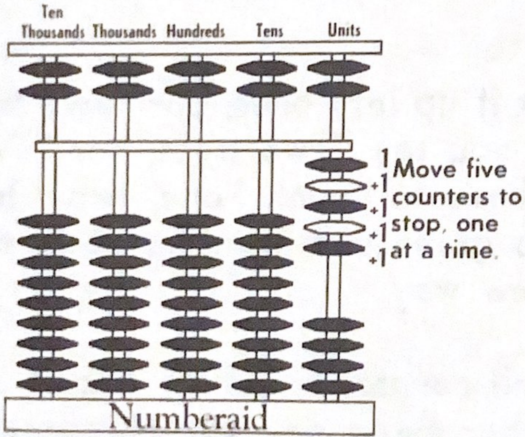
Figure 32 Four (4)



Move four counters to stop as a single group.

The Quantity Four 4

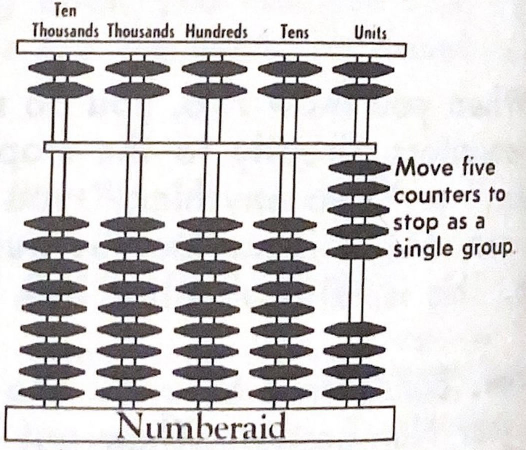
Figure 33 1+1+1+1+1=5



Move five counters to stop, one at a time.

One and One and One and One and One are Five 5

Figure 34 Five (5)

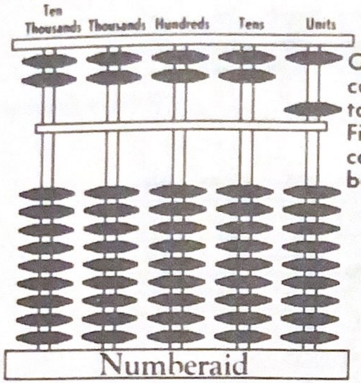


Move five counters to stop as a single group.

The Quantity Five 5

Four. Build four by adding ones, on your Numberaid (Figure 31). Show four on your Numberaid, taking four green unit-counters at a time (Figure 32).

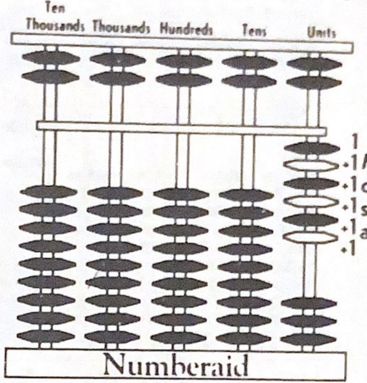
Figure 35 The Five-Unit-Counter



One Five-Unit-counter down to stop equals Five Unit-counters below stop.

The Quantity Five 5
Shown by One Five-Unit-Counter

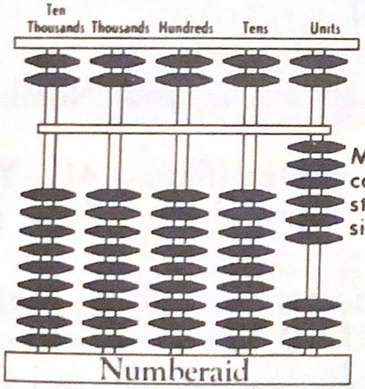
Figure 36 1+1+1+1+1+1=6



Move six counters to stop one at a time.

One and One and One 6
and One and One and One are Six

Figure 37 Six (6)



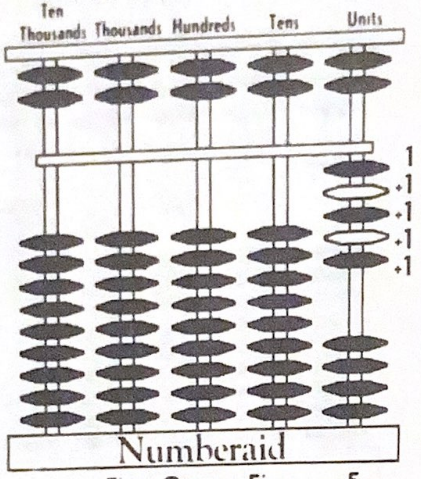
Move six counters to stop as a single group.

The Quantity Six 6

Five. Build five by adding ones (Figure 33), then show the number five (Figure 34). If we push down one solid green counter of the two above the stop on the units rod and call it a five-counter, just as you know that a nickel has the same value as five pennies, it would be a simpler way of showing five on your Numberaid. This is called changing five unit-counters to one five-counter (Figure 35).

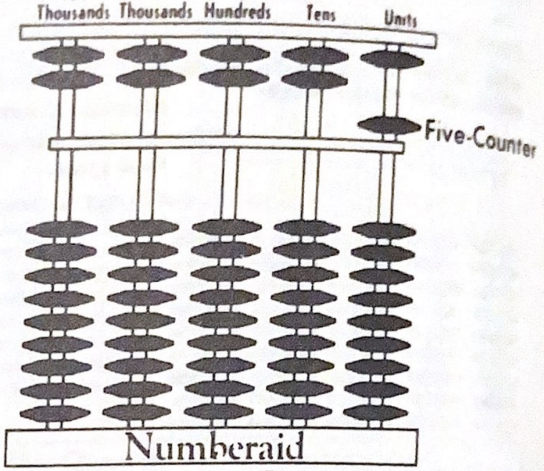
Six. Build six by adding ones (Figure 36), then show the number six (Figure 37). Then do the same for seven, eight and nine. Play Games 11, 12, 13, 14.

Figure 38
1+1+1+1+1+1+1+1+1+1=10
Part A 1+1+1+1+1=5



Five Ones = Five 5

Figure 39
1+1+1+1+1+1+1+1+1+1=10
Part B 5 Ones = 1 Five-Counter



The Quantity Five 5

Ten. To build ten requires some special instructions:

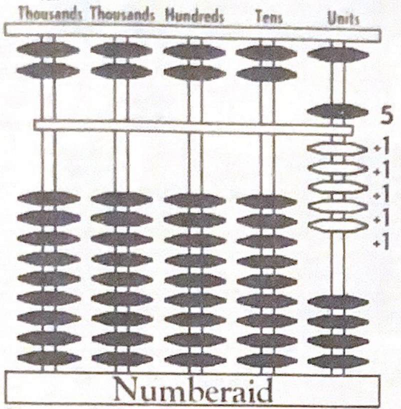
1. Clear your Numberaid.
2. Build five by adding five ones (Figure 38).
3. Change these five ones to one five-counter (Figure 39).
4. Build ten by adding five ones to the five shown on your Numberaid (Figure 40).
5. Change these five ones to a second five-counter (Figure 41). You now have two five-counters at the stop from above. They are equal to ten ones. (Figures 38-41.)
6. You cannot leave two five-counters in the units column. That is more units than the units column can hold. It can hold only nine (9). So you change the two green five-counters to one red ten-counter below the stop in the tens column (Figure 42), by moving the two green five-counters away from the stop and moving the red ten-counter up to the stop.

You have built one ten-counter from ten unit-counters by:

(a) Adding five ones to make five, and showing the five as one five-counter.

Figure 40

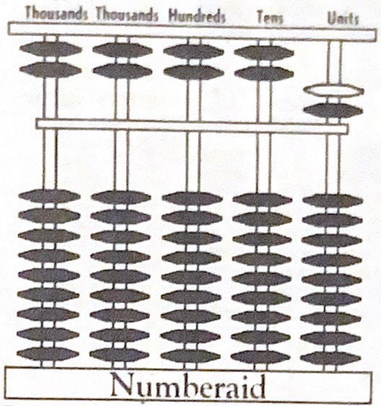
1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 10
Part C 5 + 1 + 1 + 1 + 1 + 1



Five and Five Ones are more than nine units, and cannot be shown in the units column.

Figure 41

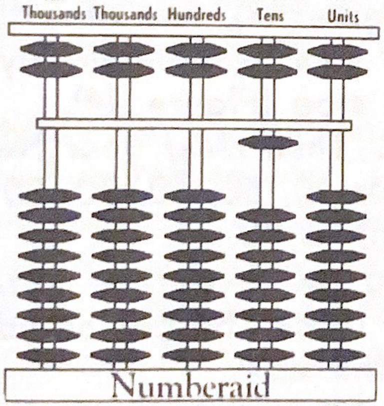
1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 10
Part D 5 + 5



Five Ones below the stop 5
are changed to a second +5
Five-Counter above the stop.

Figure 42

1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 10
Part E 5 + 5 = 10



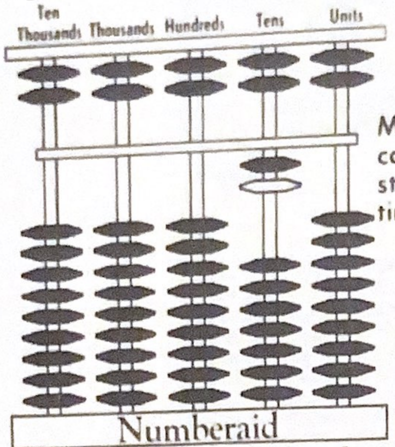
Two Fives are carried 1 0
to the Tens column
and become one red Ten-Counter.

(b) Adding five more ones to make ten ones in the units column.

(c) Changing the second five ones below the stop to one five-counter above the stop.

(d) Changing the ten ones, which are shown by two five-counters above the stop in the units column, to a ten-counter in the tens column, which is shown by one red ten-counter below the stop, since you cannot have ten ones in the units column.

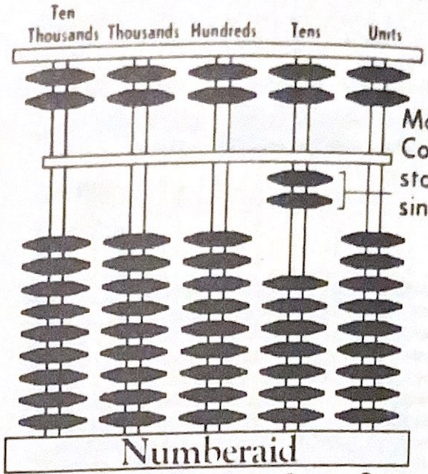
Figure 43 10 + 10 = 20



Move two ten-counters up to stop, one at a time

Ten and Ten are Twenty
1 0
+1 0
2 0

Figure 44 Twenty (20)



Move two Ten-Counters to stop as a single group

The Quantity Twenty 2 0

Twenty. You have shown that one red ten-counter is equal to ten green unit-counters. You could show twenty by two red ten-counters, which are equal to twenty green unit-counters. You can build twenty on your Numberaid by adding two red ten-counters (Figure 43), one at a time.

You can show the quantity twenty on your Numberaid by moving two red ten-counters at a time up to the stop (Figure 44).

Thirty to Ninety. Now build 30 by tens. Then clear your Numberaid and show the quantity 30. Do the same for 40, 50, 60, 70, 80 and 90. Practice showing number quantities from 10 to 90.

Hundreds. You have demonstrated (shown) that ten green unit-counters are equal to one red ten counter, and that 10 can be put on your Numberaid by moving one red ten-counter to the stop.

Ten red ten-counters are equal to one white hundred-counter, just as ten green unit-counters are equal to one red ten-counter, so you can put the quantity one hundred (100) on your Numberaid by moving one white hundred-counter to the stop (Figure 45).

Figure 45 One Hundred (100)

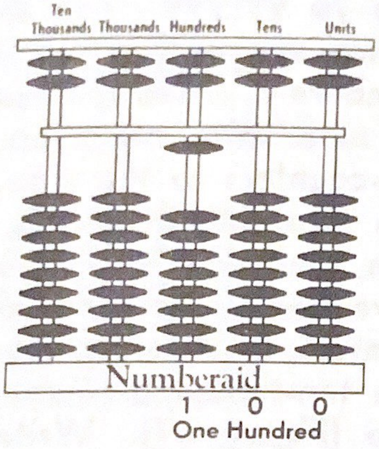
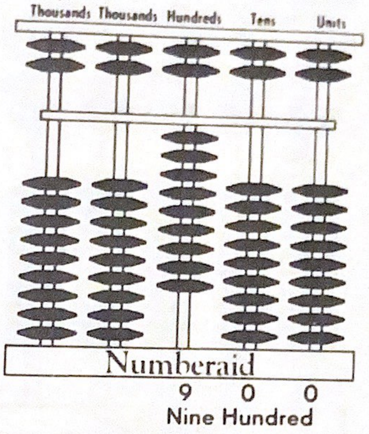


Figure 46 Nine Hundred (900)



You can show the quantities 100 to 900 by hundreds, using your Numberaid, just as easily as you can show ones or tens (Figure 46).

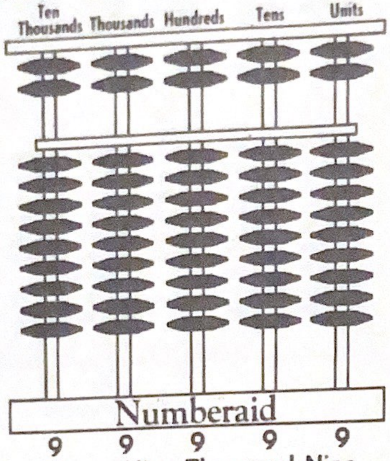
Practice counting and writing numbers from 100 to 900.

Thousands. Just as ten ten-units can be shown on your Numberaid by one white hundred-counter, so ten hundred-units can be shown on your Numberaid by one yellow thousand-counter.

Show thousands from 1,000 to 9,000 on your Numberaid, and write the numbers. You can see that it is just as easy to count by thousands as it is to count by ones or tens.

Ten Thousands. Use what you have learned to show 10,000, 20,000, 30,000, 40,000, 50,000, 60,000, 70,000, 80,000 and 90,000 on your Numberaid.

Figure 47
Ninety-Nine Thousand Nine Hundred Ninety-Nine (99,999)



Ninety-Nine Thousand Nine Hundred Ninety-Nine

One to 99,999. You are now able to show any number to 99,999 on your Numberaid. Since the number 99,999 has nine units, move up nine green unit-counters to the stop. It also has nine tens, nine hundreds, nine thousands, and nine ten-thousands, so you move nine ten-counters, nine hundred-counters, nine thousand-counters, and nine ten-thousand-counters up to the stop (Figure 47). Write 99,999 on paper as soon as you have completed it on your Numberaid.

Show the following numbers on your Numberaid and write them on paper. Keep units under units, tens under tens, and other digits in straight columns. Be sure to clear your Numberaid before starting on a new number.

1, 56, 60, 100, 125, 108, 78, 320, 1000, 2468.

8 apples, 99 trees, 101 books, 450 rabbits, 978 pennies, 1,001 beans, 6,035 mice, 4,890 men, 10,000 girls, 10,020 airplanes.

Learn how to show any number or quantity of similar objects to 99,999 on your Numberaid in a wink. Learn to write them as numbers, and to say the numbers too! If you can spell the word names of the numbers, you will be tops. Be sure to tell what they are if they have names besides the number names. This is very important, so that you know if you are talking about apples, or trees, or books.

Figure 48

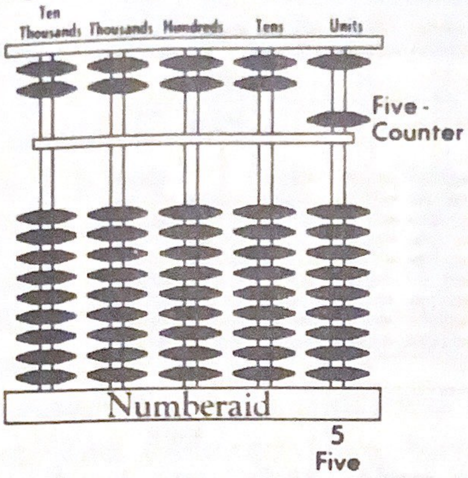
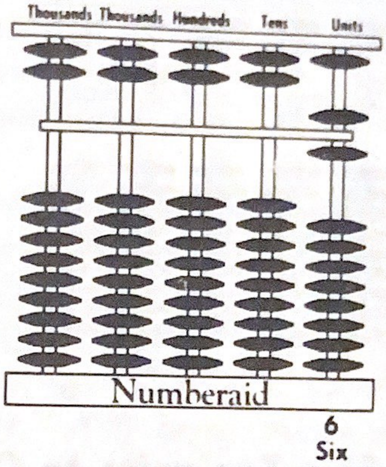


Figure 49



USE OF THE COUNTERS ABOVE THE STOP TO SHOW NUMBERS

Changing Five One-units to One Five-unit. Show the number five (5) on your Numberaid, using five green unit-counters. Five ones take up considerable space. As a short cut, which you have already demonstrated, you can use one five-counter in the place of five one-counters. You do this on your Numberaid by saying that one solid green counter **above** the stop in the units column will be a five-counter, and will have the value of (be equal to) five green unit-counters below the stop (Figure 48).

You can then show six (6) (Figure 49), using one five-counter against the stop from the two five-counters at the top of your Numberaid, and one green unit-counter against the stop from the nine green unit-counters below the stop.

How can you show 8, using the five-counter instead of five one-counters? 6? 5? 9? 7?

Figure 50 Fifty (50)

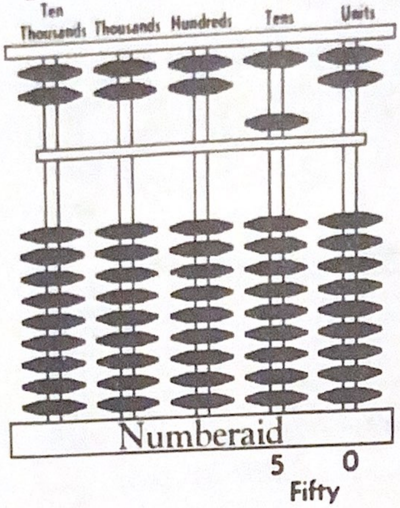
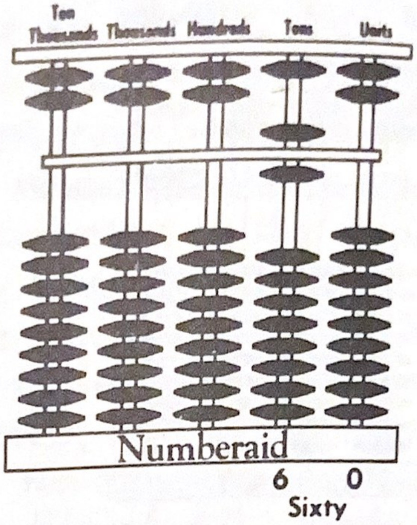


Figure 51 Sixty (60)



Changing Five Ten-units to One Fifty-unit. It is easy to show numbers in the units column when you use the five-unit-counter. You can also use one solid red counter above the stop in the tens column to replace five transparent red ten-counters below the stop in the tens column.

You can show fifty (50) (Figure 50) on your Numberaid by merely pushing one fifty-counter down to the stop.

Sixty (60) (Figure 51) would be shown by one red fifty-counter against the stop from above, and one red ten-counter against the stop from below. How would you show 70? 80? 90?

Changing Five (5) Hundred-units to a Five-hundred unit; Five (5) Thousand-units to a Five-thousand-unit; and Five (5) Ten-thousand-units to a Fifty-thousand unit. One solid white counter above the stop in the hundreds column is equal to five transparent white hundred-counters below the stop, or is equal to 500. From what you have learned about showing the numbers from 5 to 9 and from 50 to 90, how would you show the following numbers, using the five-hundred-counter: 600, 800, 700, 900, 500?

One solid yellow counter above the stop in the thousands column is equal to five (5) thousand-counters below the stop, or is equal to 5,000. How would you show the following numbers, using the five-thousand-counter? 6,000? 7,000? 8,000? 9,000?

One solid blue counter above the stop in the ten-thousands column is equal to five (5) ten-thousand-counters below the stop, or is equal to 50,000. How would you show the following numbers, using the fifty-thousand-counter: 70,000, 90,000, 80,000, 50,000?

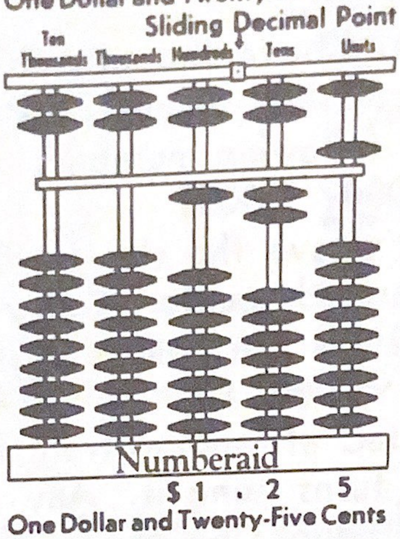
Numbers may be dictated at random from Table I. One Numberaid is used, and children take turns using it. Any error made by the child using the Numberaid, and caught by one of the other children, causes him to lose the Numberaid to the child who discovered the error. If the child using the Numberaid has shown ten successive numbers correctly, he then selects another child to take his place.

Table I

5	6	7	8	9
50	60	70	80	90
500	600	700	800	900
5,000	6,000	7,000	8,000	9,000
50,000	60,000	70,000	80,000	90,000

After some understanding and success have been achieved in showing these numbers on the Numberaid and in writing them, then dictate any numbers from one to 99,999.

Figure 52
One Dollar and Twenty-Five Cents



Dollars and Cents

The sliding decimal point at the top of the Numberaid should be put between the white and the red columns of counters. The children can practice showing and writing dollars and cents (See Figure 52). Play Game 16.

ADDITION

Problem:

$$\begin{array}{r} 0 \\ +\ 1 \\ \hline 1 \end{array}$$

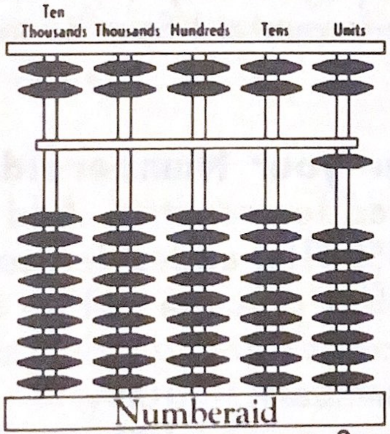
Clear your Numberaid to show zero. Add one by moving one green unit-counter up against the stop. You have added zero plus one (0 + 1). The total is one (1) (Figure 53).

Now add the following: (Be sure to clear your Numberaid before beginning a new problem.)

$$\begin{array}{rrrrrrrrrrr} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ +\ 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 50 & 250 \\ \hline \end{array}$$

You can see that if you add any number to zero, the total is the number you added. Zero plus any number equals the number.

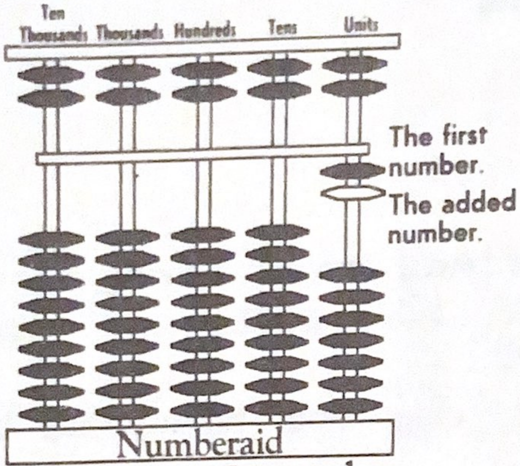
Figure 53 0+1=1



Zero and One are One

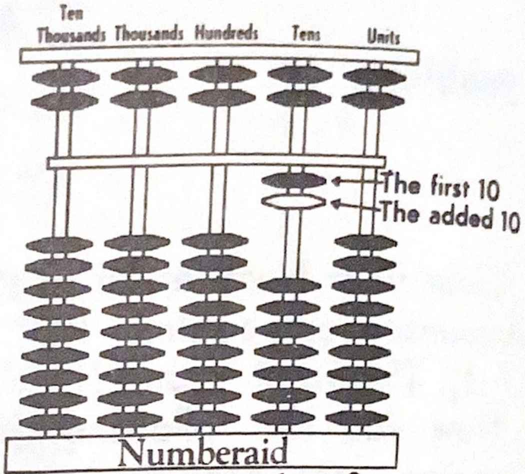
$$\begin{array}{r} 0 \\ +1 \\ \hline 1 \end{array}$$

Figure 54 1 + 1 = 2



One and One are Two
1
+1
2

Figure 55 10 + 10 = 20



Ten and Ten
are Twenty
1 0
+1 0
2 0

Problem: 1
+ 1
—

Clear your Numberaid. Show one by pushing up one green unit-counter. Add one by pushing up another green unit-counter (Figure 54). (The added unit-counter is shown in Figure 54 as an outlined counter.) Say "One and one are two."

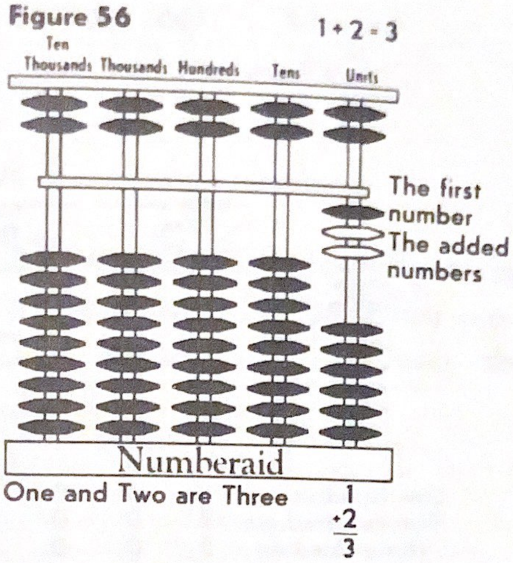
Problem: 10
+ 10
—

Clear your Numberaid. Show ten by pushing up one red ten-counter. Add ten by pushing up another red ten-counter (Figure 55). (The added ten-counter is shown in Figure 55 as an outlined counter. This practice will be followed in all illustrations.) Say "Ten and ten are twenty."

Add the following numbers:

100	1,000	10,000
100	1,000	10,000

Figure 56



Problem: 1
+ 2
—

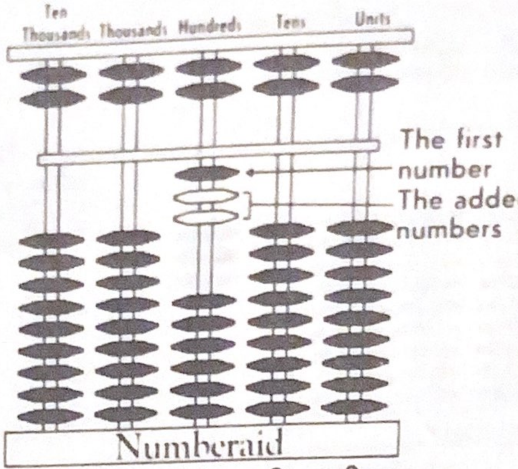
Clear your Numberaid. Show one by pushing up one green unit-counter. Add two by pushing up two green unit-counters (Figure 56). Say "One and two are three."

Problem: 2
+ 1
—

Clear your Numberaid. Show two by pushing up two green unit-counters. Add one by pushing up one more green unit-counter. Say "Two and one are three." If you add two numbers, does it make any difference which one you begin with? Of course it does not.

1 + 2 = 3, the same as 2 + 1 = 3.

Figure 57 100 + 200 = 300



One Hundred and 1 0 0
Two Hundred are +2 0 0
Three Hundred 3 0 0

Problem: 10
+ 20

Clear your Numberaid. Show ten by pushing up one red ten-counter. Add twenty by pushing up two more red ten-counters. Say "Ten and twenty are thirty."

Problem: 100
+ 200

Clear your Numberaid. Show one hundred by pushing up one white hundred-counter. Add two hundred by pushing up two more white hundred-counters (Figure 57).

Say "One hundred and two hundred are three hundred."

Problem: 1,000
+ 2,000

Clear your Numberaid. Show one thousand by pushing up one yellow thousand-counter. Add two thousand by pushing up two more yellow thousand-counters. Say "One thousand and two thousand are three thousand."

Problem: 10,000
20,000

Clear your Numberaid. Show ten thousand by pushing up one blue ten-thousand-counter. Add twenty thousand by pushing up two more blue ten-thousand-counters. Say "Ten thousand and twenty thousand are thirty thousand."

The following addition problems should be solved, adding the columns from bottom to top as well as from top to bottom.

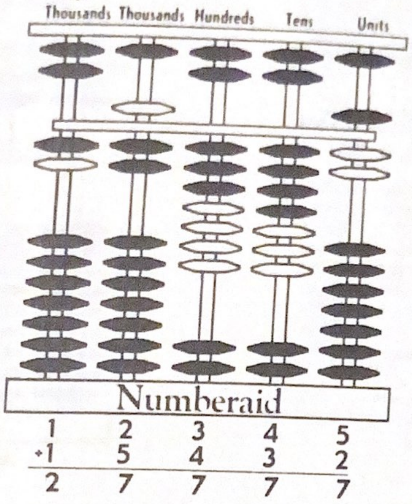
Be sure to add your columns from right to left, just as you do with pencil and paper. Play Games 15. 16, 17, 18.

1 + 3	10 30	100 300	1,000 3,000	10,000 30,000
1 4	10 40	100 400	1,000 4,000	10,000 40,000
1 5	10 50	100 500	1,000 5,000	10.000 50,000
1 6	10 60	100 600	1,000 6,000	10,000 60,000
1 7	10 70	100 700	1,000 7,000	10,000 70,000
1 8	10 80	100 800	1,000 8,000	10,000 80,000

12.345 54,321	12,345 32,154	12,345 15,432	6,789 1,000
1,234 4,321	23,451 15,432	6,789 3,210	

Figure 58

12,345 + 15,432 = 27,777



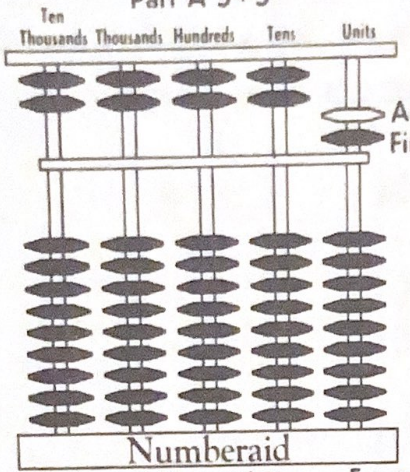
Problem: 12,345
+ 15,432

Clear your Numberaid. Show the number 12,345 on your Numberaid by pushing one green five-unit-counter, four red ten-counters, three white hundred-counters, two yellow thousand-counters and one blue ten-thousand-counter to the stop. This number is shown in Figure 58 by the black counters.

Add the number 15,432 by pushing two green unit-counters, three red ten-counters, four white hundred-counters, one yellow five-thousand-counter and one ten-thousand-counter to the stop. This number is shown in Figure 58 by the outlined counters.

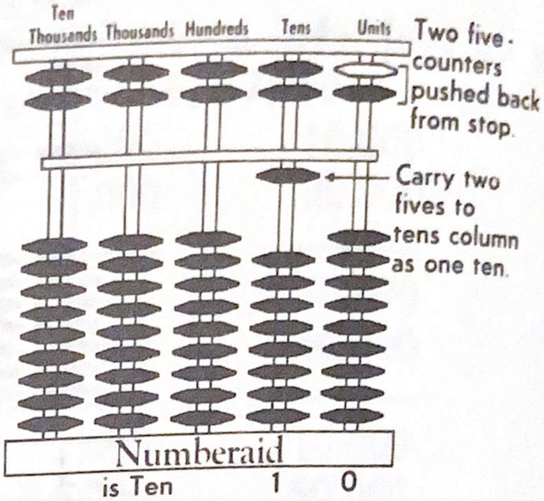
The total sum 27,777 can be read directly.

Figure 59 5 + 5 = 10
Part A 5 + 5



The sum of Five and Five 5 +5

Figure 59 5 + 5 = 10
Part B "Carry" one Ten


Ten Thousands
Thousands
Hundreds
Tens
Units
Two five-counters pushed back from stop.
Carry two fives to tens column as one ten.
Numberaid

is Ten 1 0

Carrying

Problem: 5
+ 5

Clear your Numberaid. Show the number five by pushing one green **five-counter** down to the stop (Part a, Figure 59).

Push a second green five-counter down to the stop (Part a, Figure 59).

You cannot show a number greater than nine in the units column, as shown by two five-counters at the stop from the top. They must be carried to the next column and changed (transformed) into a ten-counter which has the same value as the two five-counters above the stop. This is called "Carrying ten" or "Carrying." Now push the two five-counters away from the stop, and push one red ten-counter up to the stop (Part b, Figure 59) saying "The sum of five and five is ten."

Figure 60 50 + 50 = 100
Part A "Carry" one Hundred

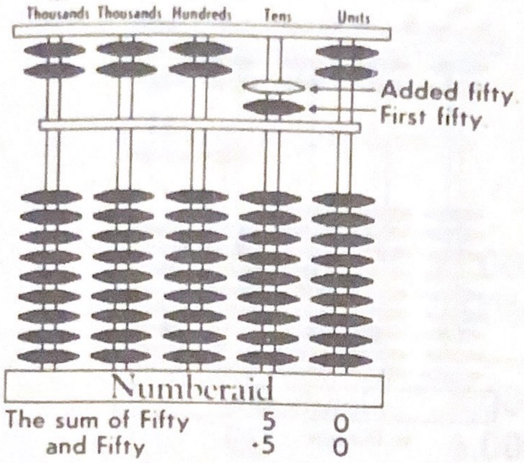
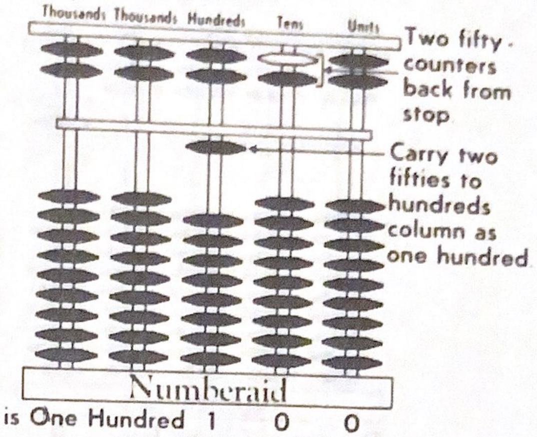


Figure 60 50 + 50 = 100
Part B 50 + 50



Ten Thousands
Thousands
Hundreds
Tens
Units
Two fifty-counters back from stop
Carry two fifties to hundreds column as one hundred
Numberaid
is One Hundred
1
0
0

Problem: 50
+ 50
—

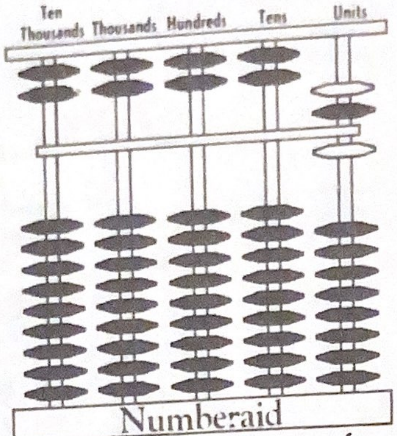
Clear your Numberaid. Show the number fifty by pushing one red **fifty-counter** down to the stop (Part a, Figure 60). Add fifty by pushing a second red fifty-counter down to the stop (Part a, Figure 60). To carry into the next column, push the two red fifty-counters away from the stop and push one white hundred-counter up to the stop (Part b, Figure 60), saying "The sum of fifty and fifty is one hundred."

Problem: 500
+ 500
—

Clear your Numberaid. Show the number five hundred by pushing one white five-hundred-counter down to the stop. Add five hundred by pushing a second white five-hundred-counter down to the stop. To carry into the next column, push up one yellow thousand-counter to the stop, and push the two white five-hundred-counters away from the stop, saying "The sum of five hundred and five hundred is one thousand."

Figure 61 6 + 5 = 11

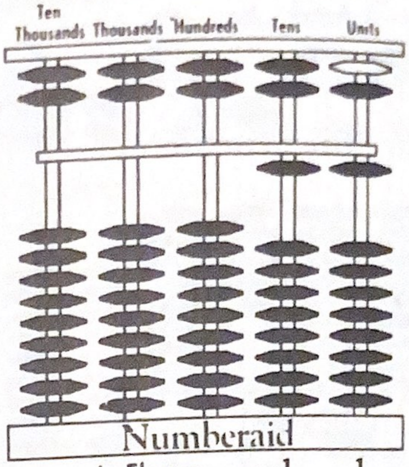
Part A 6 + 5



The sum of Six and Five $\begin{array}{r} 6 \\ +5 \end{array}$

Figure 61 6 + 5 = 11

Part B "Carry" one Ten



is Eleven 1 1

Problem: $\begin{array}{r} 6 \\ +\ 5 \\ \hline \end{array}$

Clear your Numberaid. Show five on your Numberaid, using one five-counter (Part a, Figure 61). Show six more on your Numberaid, using a five-counter and a unit-counter (Part a, Figure 61). Carry the 10 by pushing two green five-counters away from the stop, and pushing one red ten-counter up to the stop (Part b, Figure 61), saying "The sum of five and six is eleven."

Now do the following addition problems:

$$\begin{array}{rrrrrr} 6 & 60 & 600 & 6{,}000 & 7 & 700 \\ +\ 5 & 50 & 500 & 5{,}000 & 5 & 500 \\ \hline \end{array}$$

$$\begin{array}{rrrr} 80 & 8{,}000 & 9 & 900 \\ 50 & 5{,}000 & 5 & 500 \\ \hline \end{array}$$

$$\begin{array}{r} 60 \\ +\ 60 \\ \hline \end{array} \qquad \begin{array}{r} 6{,}000 \\ 6{,}000 \\ \hline \end{array} \qquad \begin{array}{r} 7 \\ 6 \\ \hline \end{array} \qquad \begin{array}{r} 600 \\ 700 \\ \hline \end{array}$$

$$\begin{array}{r} 80 \\ 60 \\ \hline \end{array} \qquad \begin{array}{r} 8{,}000 \\ 6{,}000 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ 6 \\ \hline \end{array} \qquad \begin{array}{r} 900 \\ 600 \\ \hline \end{array}$$

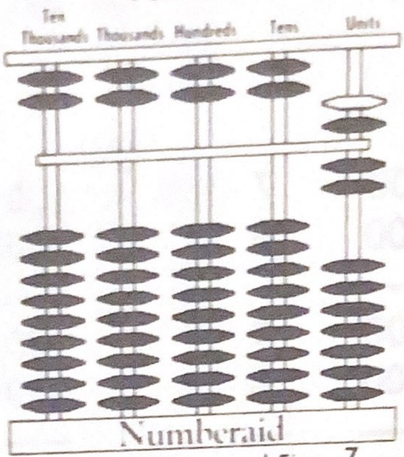
$$\begin{array}{r} 70 \\ 70 \\ \hline \end{array} \qquad \begin{array}{r} 7{,}000 \\ 7{,}000 \\ \hline \end{array} \qquad \begin{array}{r} 7 \\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 700 \\ 800 \\ \hline \end{array}$$

$$\begin{array}{r} 70 \\ 90 \\ \hline \end{array} \qquad \begin{array}{r} 7{,}000 \\ 9{,}000 \\ \hline \end{array} \qquad \begin{array}{r} 8 \\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 800 \\ 800 \\ \hline \end{array}$$

$$\begin{array}{r} 80 \\ 90 \\ \hline \end{array} \qquad \begin{array}{r} 8{,}000 \\ 9{,}000 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ 9 \\ \hline \end{array} \qquad \begin{array}{r} 900 \\ 900 \\ \hline \end{array}$$

Figure 62 55+27 82

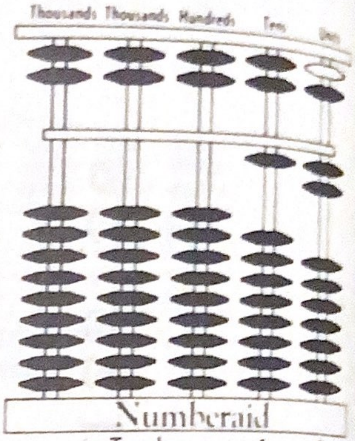
Part A 7+5



The sum of Seven and Five 7
+5

Figure 62 55+27 82

Part B 7+5 12

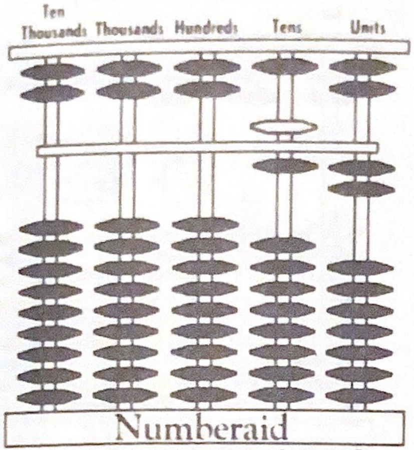


is Twelve 1 2

Problem: 55
+ 27

(Part a, Figure 62). Clear your Numberaid. Add 7 +5.
(Part b, Figure 62). Carry the 10.

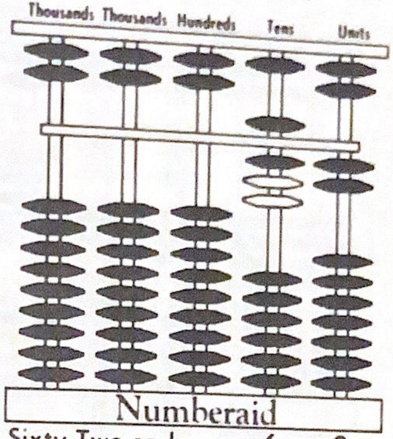
Figure 62 55 + 27 = 82
Part C 12 + 50 = 62



Twelve and Fifty
are Sixty-Two

1 2
+5 0
6 2

Figure 62 55 + 27 = 82
Part D 62 + 20 = 82



Sixty-Two and
Twenty are
Eighty-Two

6 2
+2 0
8 2

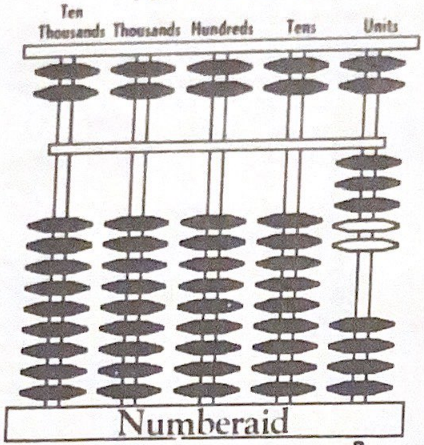
(Part c, Figure 62). Add 50.
(Part d, Figure 62). Add 20.

Do the following addition problems:

65	55	27	75
+ 38	29	13	19
—	—	—	—

(For supplementary problems, use arithmetic textbook.)

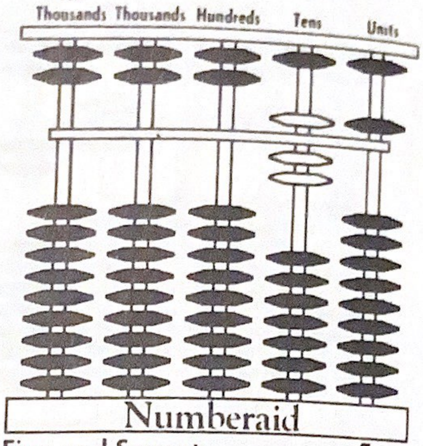
Figure 63 72 + 43 = 115
Part A 3 + 2 = 5



Three and Two are Five

3
+2
5

Figure 63 72 + 43 = 115
Part B 5 + 70 = 75



Five and Seventy are Seventy-Five

5
+7 0
7 5

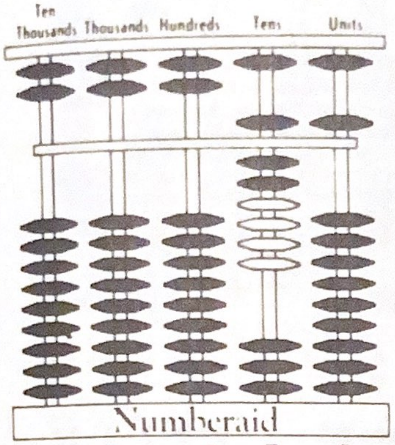
Problem: 72
+ 43

(Part a, Figure 63). Add 3 + 2.
(Part b, Figure 63). Add 5 + 70.
(Part c, Figure 63). Add 40.
(Part d, Figure 63). Exchange five ten-counters for one fifty-counter.
(Part e, Figure 63). Carry the 100 into hundreds place.

If you know how to do these addition problems, you can, with practice, do any whole number addition problems containing two digits, whose totals are not greater than 99,999. You need practice to gain speed and accuracy.

Figure 63 72 + 43 115

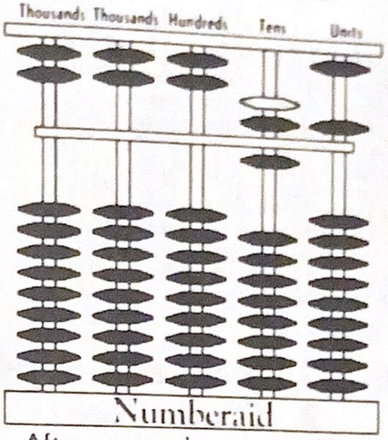
Part C 75 + 40



The sum of Seventy-Five 7 5
and Forty +4 0

Figure 63 72 + 43 115

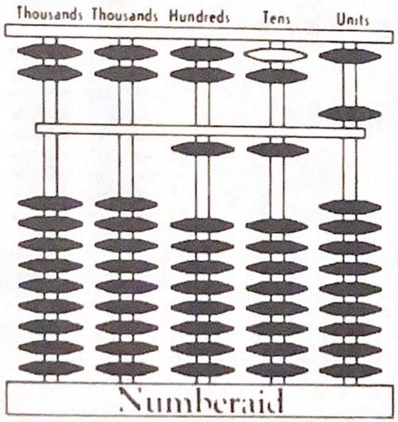
Part D Change 5 tens for one fifty and carry one hundred



After you exchange one fifty-counter for five ten-counters and carry one hundred

Figure 63 72 + 43 115

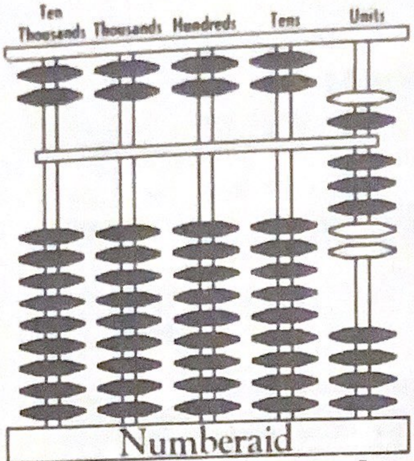
Part E



1 1 5

is One Hundred-Fifteen

Figure 64 9 + 8 + 7 + 8 = 32
Part A 8 + 7



The sum of Eight and Seven 8 +7

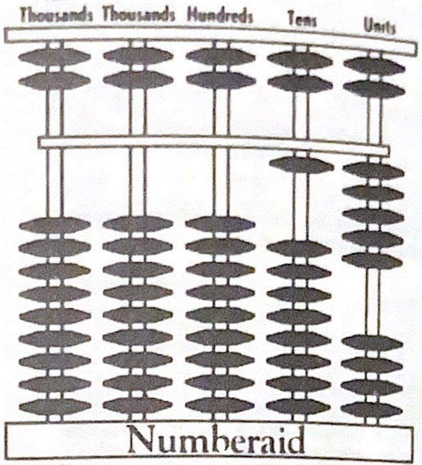
Problem: 9 (Part a, Fig. 64).
8 Clear your
7 Numberaid.
8 Add 8 + 7.
— (Part b, Fig. 64). Carry the 10 into the tens column.
(Part c, Fig. 64). Add 8.

(Part d, Fig. 64). Carry a second 10 into the tens column.

(Part e, Fig 64). Add 9.

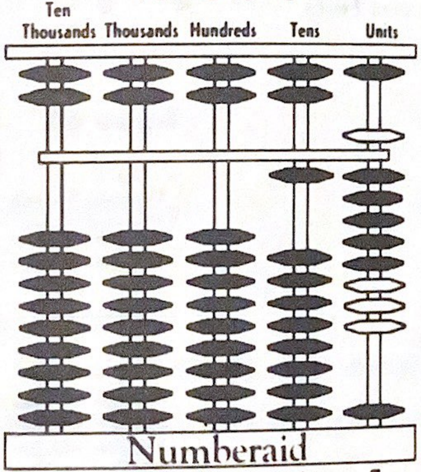
(Part f, Fig. 64). Carry a third 10 into the tens column.

Figure 64 9 + 8 + 7 + 8 = 32
Part B 8 + 7 = 15



is Fifteen 1 5

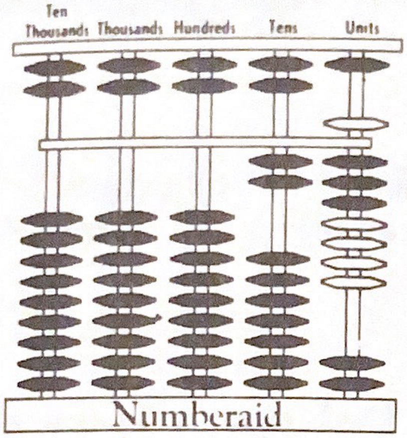
Figure 64 9 + 8 + 7 + 8 = 32
Part C 15 + 8



Fifteen and Eight 1 5 + 8

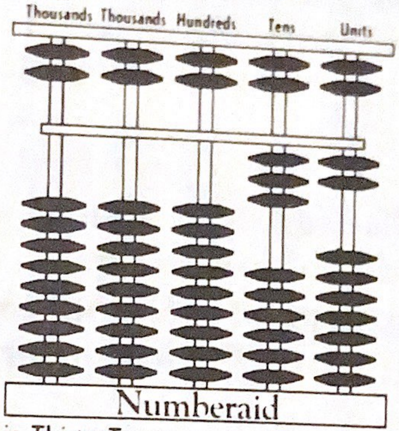
Now using this pattern, do the addition problems on the following page.

Figure 64 9 + 8 + 7 + 8 = 32
Part E 23 + 9



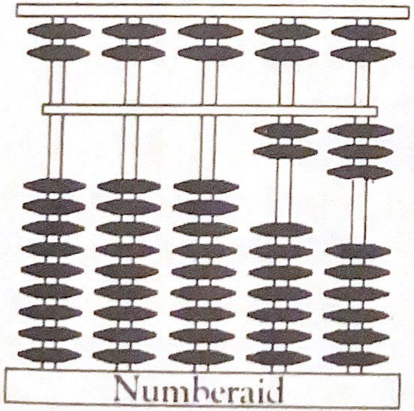
The sum of Twenty-Three and Nine 2 3
+ 9

Figure 64 9 + 8 + 7 + 8 = 32
Part F 23 + 9 = 32



is Thirty-Two 3 2

Figure 64 9 + 8 + 7 + 8 = 32
Part D 15 + 8 = 23



are Twenty-Three 2 3

923
+ 389

6,879
3,121

43,576
67,535

7,000
6,000
9,000
8,000

4,000
7,000
9,000
3,000
8,000

800
700
500
900
600
700

20
90
50
30
70
60

MULTIPLICATION

Clear your Numberaid. Add eight ones. When you write this addition on paper it looks like this:

$$\begin{array}{r} 1 \\ \times\ 8 \\ \hline 8 \end{array}$$

it is called Multiplication.

Multiplication is a special case of addition in which the numbers added (addends) are all equal.

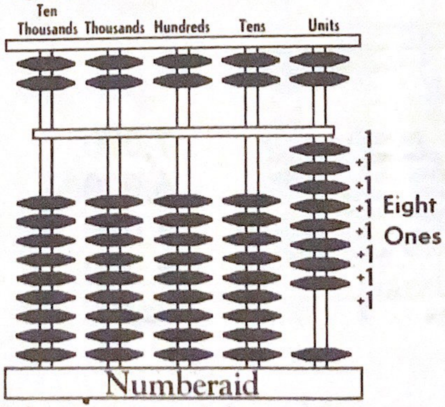
The problem

$$\begin{array}{rl} 1 & \text{Multiplicand} \\ \times\ 8 & \text{Multiplier} \\ \hline 8 & \text{Product} \end{array}$$

tells that one is to be added eight times (Figure 65).

One is the multiplicand (the number to be added). Eight, the multiplier, tells how many times the multiplicand is to be added. Eight, the product, is the sum of the addition of the multiplicand the number of times indicated by the multiplier.

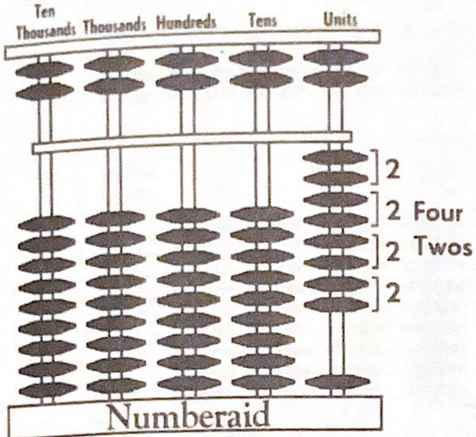
Figure 65 8 × 1 = 8



Eight times one is Eight
Eight ones = Eight
One added eight times is eight

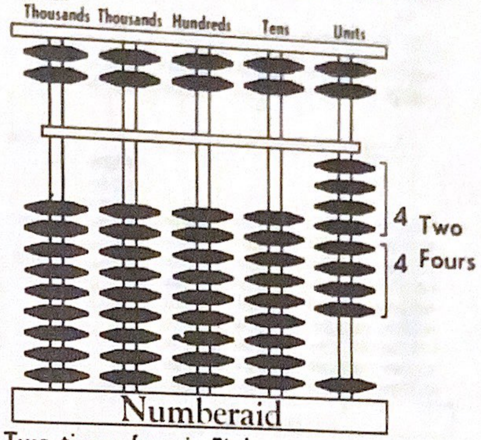
$$\begin{array}{r} 1 \\ \times 8 \\ \hline 8 \end{array}$$

Figure 66 4 × 2 = 8



Four times two is Eight 2
Four twos = Eight ×4
Two added four times is Eight 8

Figure 67 2 × 4 = 8



Two times four is Eight 4
Two fours = Eight ×2
Four added two times is Eight 8

To total eight, twos could be added. You would then begin with two, add two to make four, add two to make six, and finally, add two to make eight (Figure 66).

This addition, written as a multiplication problem, would look like this:

$$\begin{array}{r} 2 \\ \times\ 4 \\ \hline 8 \end{array}$$

It says four twos are to be added.

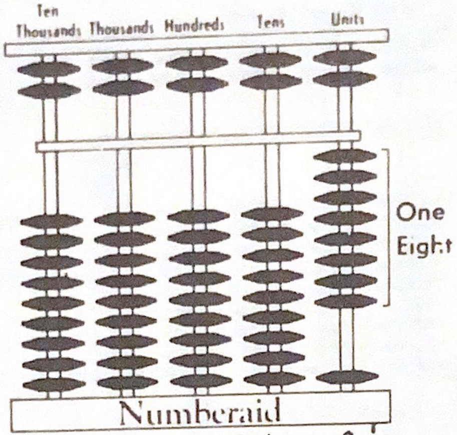
To total eight, fours could be added two times.

Clear your Numberaid. Begin with four. Add four more. The total is eight (Figure 67).

You can find the total more quickly because there are less figures to deal with. As a multiplication problem, adding two fours would be written:

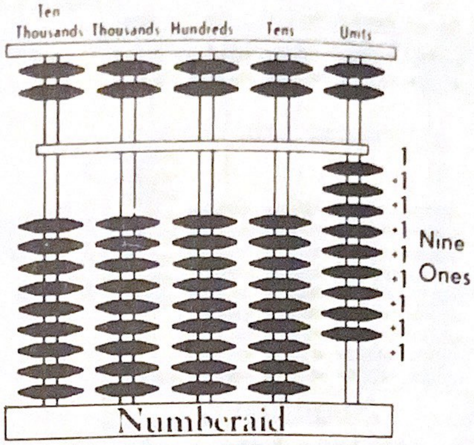
$$\begin{array}{rl} 4 & \text{Multiplicand} \\ \times\ 2 & \text{Multiplier} \\ \hline 8 & \text{Product} \end{array}$$

Figure 68 1 × 8 8



One times eight is Eight 8
One eight Eight ×1
Eight added once is Eight 8

Figure 69 9 × 1 9



Nine times one is Nine 1
Nine Ones Nine ×9
One added nine times is Nine 9

Remember, in multiplication the second number, or the multiplier, tells how many times you should add the first number, or the multiplicand.

Did you notice that the product is the same whether you multiplied 2 x 4 or 4 x 2?

There is still another way of finding numbers that total eight. **Clear your Numberaid.** Show one eight on it (Figure 68).

Written as a multiplication problem, it would be

$$\begin{array}{rl} 8 & \text{Multiplicand} \\ \times\ 1 & \text{Multiplier} \\ \hline 8 & \text{Product} \end{array}$$

The second number, or multiplier, tells you to add eight once.

Do the following multiplication problems by addition. The first problem is shown in Figure 69.

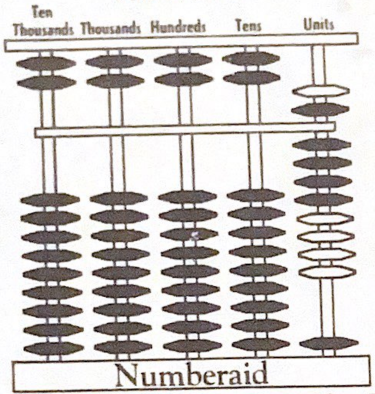
$$\begin{array}{rrrrrrrrr}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
\underline{\times 9} & \underline{8} & \underline{7} & \underline{6} & \underline{5} & \underline{4} & \underline{3} & \underline{2} & \underline{1}\\
2 & 2 & 2 & 2 & & & & & \\
\underline{1} & \underline{2} & \underline{3} & \underline{4} & & & & & \\
3 & 3 & 3 & & & & & & \\
\underline{1} & \underline{2} & \underline{3} & & & & & & \\
4 & 4 & & & & & & & \\
\underline{1} & \underline{2} & & & & & & & \\
5 & 6 & 7 & 8 & 9 & & & & \\
\underline{1} & \underline{1} & \underline{1} & \underline{1} & \underline{1} & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
\underline{2} & \underline{3} & \underline{4} & \underline{5} & \underline{6} & \underline{7} & \underline{8} & \underline{9} &
\end{array}$$

If you add zero (nothing) two times, you have nothing. So you can understand that 0 x any number is always 0.

If you add a number no times, you have nothing, or zero. So you can understand that any number multiplied by zero is zero.

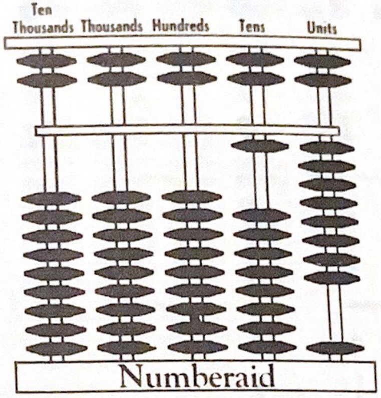
$$\begin{array}{rrrrrrrrrr}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9\\
\underline{\times 0} & \underline{0} & \underline{0} & \underline{0} & \underline{0} & \underline{0} & \underline{0} & \underline{0} & \underline{0} & \underline{0}
\end{array}$$

Figure 70 2 × 9 = 18
Part A 2 × 9



Two times nine 9 or 9 Two
Two nines x2 +9 Nines
Nine added two times

Figure 70 2 × 9 = 18
Part B Carry one ten



is Eighteen 9
= Eighteen +9
is Eighteen 1 8

Do the following problems by addition. Remember, the second number, the multiplier, tells you how many times you must add the first number, the multiplicand, to get the product. Do the addition on your Numberaid. Write the addition and the product on your paper. Try to remember the products. You will use them over and over.

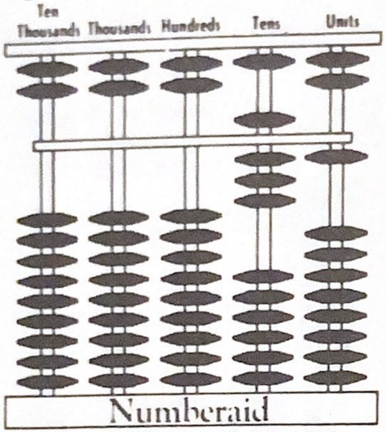
The product of the first problem in the last row would look like Figure 70, Part a, on your Numberaid before you "carry" the ten. The final product is shown in Figure 70, Part b. Written as you find it in arithmetic, it would look like this:

$$\begin{array}{r} 9 \\ \times\ 2 \\ \hline 18 \end{array}$$

2	2	2	2	2			6	6	6	6	6	6	6	6
x 5	6	7	8	9			2	3	4	5	6	7	8	9
—	—	—	—	—			—	—	—	—	—	—	—	—
3	3	3	3	3	3		7	7	7	7	7	7	7	7
4	5	6	7	8	9		2	3	4	5	6	7	8	9
—	—	—	—	—	—		—	—	—	—	—	—	—	—
4	4	4	4	4	4	4	8	8	8	8	8	8	8	8
3	4	5	6	7	8	9	2	3	4	5	6	7	8	9
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5	5	5	5	5	5	5	9	9	9	9	9	9	9	9
3	4	5	6	7	8	9	2	3	4	5	6	7	8	9
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

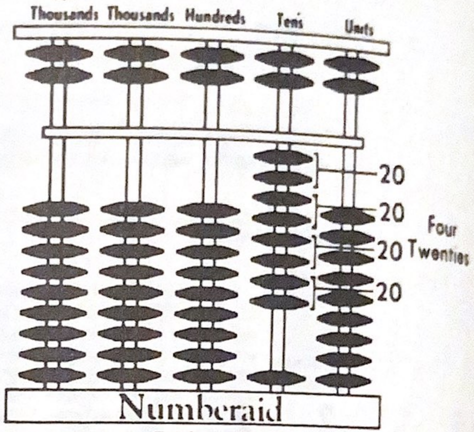
Does 2 x 5 produce the same product as 5 x 2? If so, can you say that in any multiplication problem, you can change the order of the multiplicand and multiplier without changing the product? Would such a discovery be useful in working with numbers? Why? Play Games 19 and 20.

Figure 71 9 × 9 = 81



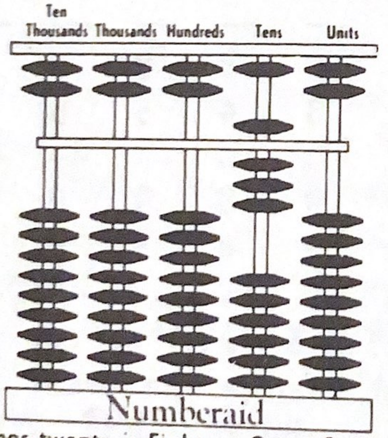
Nine times nine is Eighty-One 9
Nine nines = Eighty-One ×9
8 1

Figure 72 4 × 20 = 80



Four times twenty is Eighty 2 0
Four twenties = Eighty ×4
Twenty added four times is Eighty 8 0

Figure 73 4 × 20 = 80



Four times twenty is Eighty 2 0
Four twenties = Eighty ×4
8 0

If you remember the products of the problems you worked out by addition on the previous page, you can use them in solving problems. It is much easier and quicker to know these products and put them directly on your Numberaid without having to figure them out by addition every time you need them. Practice putting the products of the one-digit multiplication problems on the previous pages on your Numberaid without working them out by addition.

The product of the last problem 9 x 9, put on your Numberaid from memory, is shown in Figure 71.

Dictate ten multiplication problems from those given on previous pages. Have the children work out the products by addition on their Numberaids, and write the answers. Dictate the same ten multiplication problems, in a different order. Have the children put only the answers on their Numberaids, and write them on their paper. The one who has the largest number of correct answers wins the game.

Do the following problems by addition. The product of multiplication of the fourth problem in the first row is shown in Figure 72.

After you have done these problems by addition, you should be able to put the products directly on your Numberaid (Figure 73).

Multiplication is easy if you think as you work on your Numberaid.

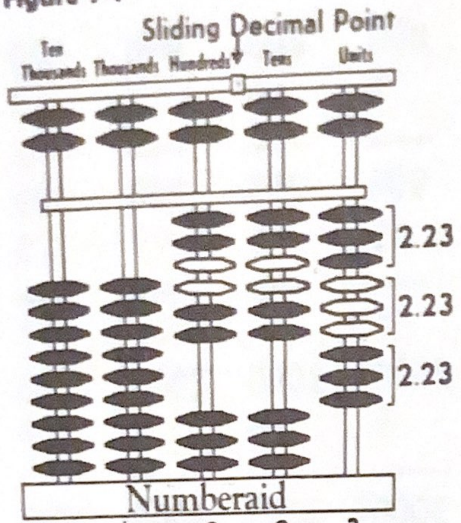
20	20	20	20	20	20	20	20	20
x 1	2	3	4	5	6	7	8	9
—	—	—	—	—	—	—	—	—
30	30	30	30	30	30	30	30	30
1	2	3	4	5	6	7	8	9
—	—	—	—	—	—	—	—	—
40	40	40	40	40	40	40	40	40
1	2	3	4	5	6	7	8	9
—	—	—	—	—	—	—	—	—
50	50	50	50	50	50	50	50	50
1	2	3	4	5	6	7	8	9
—	—	—	—	—	—	—	—	—
60	60	60	60	60	60	60	60	60
1	2	3	4	5	6	7	8	9
—	—	—	—	—	—	—	—	—

70 x 1 —	70 2 —	70 3 —	70 4 —	70 5 —	70 6 —	70 7 —	70 8 —	70 9 —
80 1 —	80 2 —	80 3 —	80 4 —	80 5 —	80 6 —	80 7 —	80 8 —	80 9 —
90 1 —	90 2 —	90 3 —	90 4 —	90 5 —	90 6 —	90 7 —	90 8 —	90 9 —

200 1 —	200 2 —	200 3 —	200 4 —	200 5 —	200 6 —	200 7 —	200 8 —	200 9 —
300 1 —	300 2 —	300 3 —	300 4 —	300 5 —	300 6 —	300 7 —	300 8 —	300 9 —
400 1 —	400 2 —	400 3 —	400 4 —	400 5 —	400 6 —	400 7 —	400 8 —	400 9 —
500 1 —	500 2 —	500 3 —	500 4 —	500 5 —	500 6 —	500 7 —	500 8 —	500 9 —
600 1 —	600 2 —	600 3 —	600 4 —	600 5 —	600 6 —	600 7 —	600 8 —	600 9 —
700 1 —	700 2 —	700 3 —	700 4 —	700 5 —	700 6 —	700 7 —	700 8 —	700 9 —

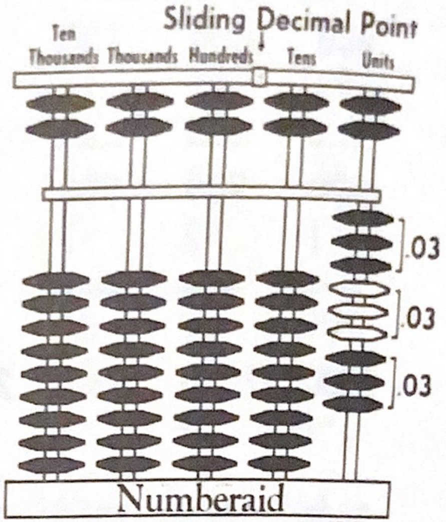
800 x 1	800 2	800 3	800 4	800 5	800 6	800 7	800 8	800 9
900 1	900 2	900 3	900 4	900 5	900 6	900 7	900 8	900 9
2000 1	2000 2	2000 3	2000 4	2000 5	2000 6	2000 7	2000 8	2000 9
3000 1	3000 2	3000 3	3000 4	3000 5	3000 6	3000 7	3000 8	3000 9
4000 1	4000 2	4000 3	4000 4	4000 5	4000 6	4000 7	4000 8	4000 9
5000 1	5000 2	5000 3	5000 4	5000 5	5000 6	5000 7	5000 8	5000 9
6000 1	6000 2	6000 3	6000 4	6000 5	6000 6	6000 7	6000 8	6000 9
7000 1	7000 2	7000 3	7000 4	7000 5	7000 6	7000 7	7000 8	7000 9
8000 1	8000 2	8000 3	8000 4	8000 5	8000 6	8000 7	8000 8	8000 9
9000 1	9000 2	9000 3	9000 4	9000 5	9000 6	9000 7	9000 8	9000 9

Figure 74 3 × 2.23 = 6.69



Three times Two and 2 . 2 3
Twenty-Three Hundredths is ×3 . 0 0
Six and Sixty-Nine Hundredths 6 . 6 9

Figure 75 3 × 2.23 = 6.69
Part A 3 × .03 = .09



Three .03's = .09

.03
×3.00
.09

Problem: 2.23
x 3

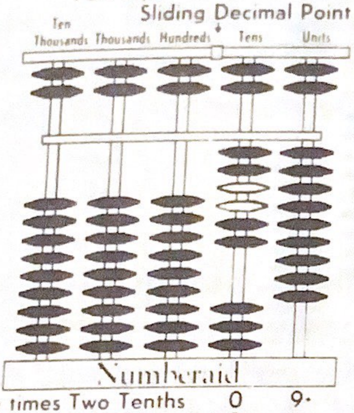
This problem in multiplication tells you to add 2.23 three times (Figure 74). Place the sliding decimal point on your Numberaid between the tens and hundreds columns. Now you will use the white counters for units, the yellow counters for tens, etc.

You can do this same problem the pencil and paper way (Figure 75, Parts a, b, and c).

Problem: 2.23
x 3

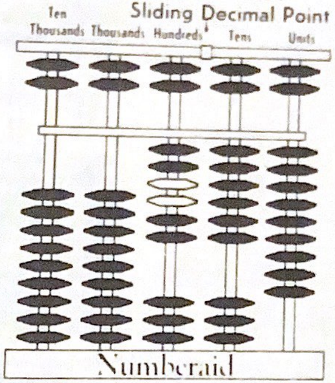
The problem says: (a) Take .03 three times (3 x .03) = .09
(b) Take .2 three times (3 x .2 = .60) = .69
(c) Take 2 three times (3 x 2 = 6.00) = 6.69

Figure 75 3 × 2 23 6 69
Part B (3 × 2) · 09 69



Three times Two Tenths 0 9·
plus Nine Hundredths 2
equals ·3 0 0
Sixty Nine Hundredths 6 9

Figure 75 3 × 2 23 6 69
Part C (3 × 2) · 69 6 69



Three times Two plus 6 9·
Sixty-Nine Hundredths 2 0 0
equals Six and ·3 0 0
Sixty-Nine Hundredths 6 6 9·

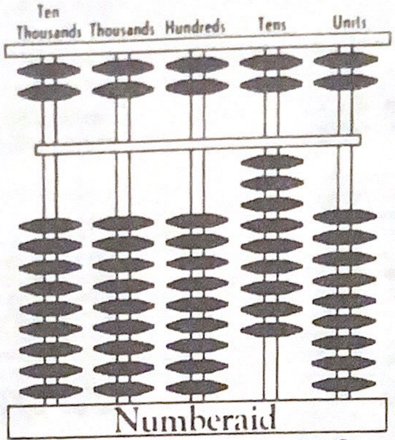
Both ways are good ways, and should be understood. When working with pencil and paper, the problem is usually written like this:

$$\begin{array}{r} 2.23 \\ \times\ 3 \\ \hline .09 \\ .60 \\ 6.00 \\ \hline 6.69 \end{array}$$

Now work the following multiplication problems by both methods using one method to check the other:

6.75	.05	.35	12.67	2.53
x 5	3.	6.	4.	7.

Figure 76 10 × 9 = 90



Ten times Nine
equals Ninety
Ten 9's = 90

9
×10
90

Find the products of the following multiplication problems by addition. The product of the ninth problem is shown in Figure 76.

1	2	3	4	5	6	7	8	9
x 10	10	10	10	10	10	10	10	10
10	20	30	40	500	600	700		
10	10	10	10	10	10	10		
8000	8000	35	457	9325				
10	10	10	10	10				

Examine the products of all problems with a multiplier of 10. Can you say that you can multiply any number by 10 by adding one zero to the number? Can you multiply any whole number by 10 by writing it one column to the left of its regular position? Of course you can.

Now show the following products on your Numberaid without adding, merely by moving the multiplicands one column to the left of where they would normally be placed on your Numberaid.

$$\begin{array}{rrrr} 23 & 976 & 677 & 5435 \\ \times\ 10 & 10 & 10 & 10 \\ \hline \end{array}$$

What do you suppose would happen if you used the multiplier 100 instead of 10? How many zeros would you add to multiply any number by 100 (which can be written 10^2)? How many places would you move it to the left on your Numberaid? What about multiplication by 1,000 (which can be written 10^3)? By any power of ten (10^4) (10^5)? Can you make up a rule which will fit multiplication of any number by ten or any power of ten?

Show $1.25 on your Numberaid. What would you do to multiply it by 10? What would be the product? What would you do to multiply it by 100? by 1000? What would be the products?

Do the following problems:

One bar of soap costs $ .15? How much would ten bars cost? One hundred bars?

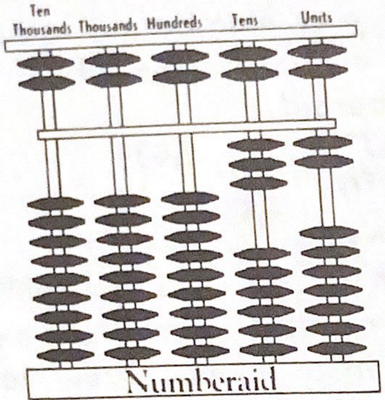
If one pound of meat costs $ .85, how much would ten pounds cost?

Find the product:

$$\begin{array}{rrrrr} 1.35 & .05 & .15 & .69 & \$23.46 \\ 10. & 100. & 10. & 1000. & 10. \\ \hline \end{array}$$

Figure 77 21 × 32 672

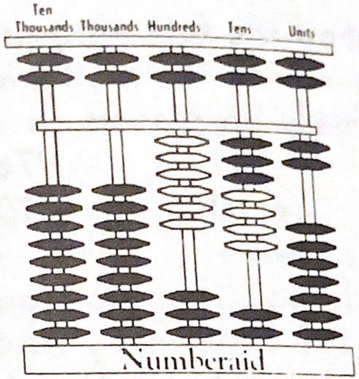
Part A 1 × 32 = 32



One Thirty-Two 3 2
equals Thirty-Two ×1
One 32 = 32 3 2

Figure 77 21 × 32 672

Part B (2 × 320) · 32 672



Two times Three Hundred 3 2·
Twenty plus Thirty-Two 3 2 0
equals ×2
Six Hundred Seventy-Two 6 7 2

Multiplication by a Two-digit Multiplier

Problem: 32 (By addition) (Additive Method)
x 21 This problem says, add 32 one time (Figure 77,
— Part a) Then multiply 32 by 20 moving it one column to the left, and add 320 twice (Figure 77, Part b).

You can do this problem another way. We will call it cumulative multiplication, or the pencil and paper way.

Problem: 32
x 21 This way the problem says:
—

(a) Take 2 one time (2 x 1) = 2 (Figure 78, Part a).
(b) Add 30 one time (30 x 1 = 30) = 32 (Figure 78, Part a).
(c) Add 2 twenty times (2 x 20 = 40) = 72 (Figure 78, Part b).
(d) Add 30 twenty times (30 x 20 = 600) = 672 (Figure 78, Part b).

The final product should be 672, the same as you found when you did the problem the first way.

Figure 78 21 × 32 = 672
Part A (30 × 1) + (2 × 1) = 32

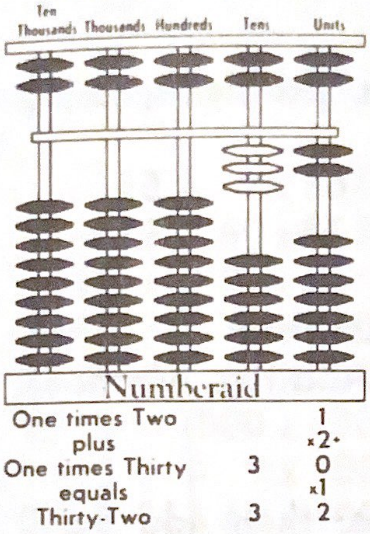
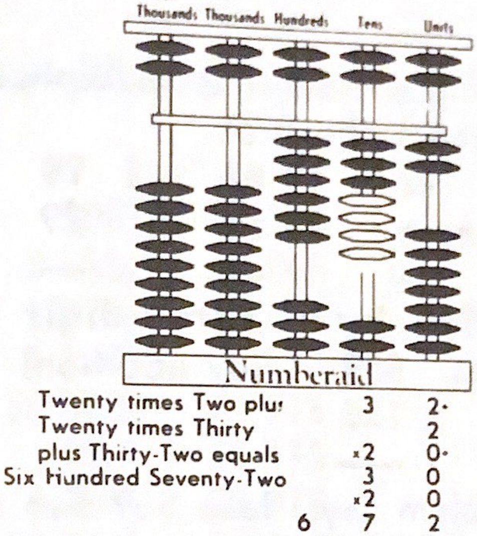


Figure 78 21 × 32 = 672
Part B (20 × 30) + (20 × 2) + 32 = 672



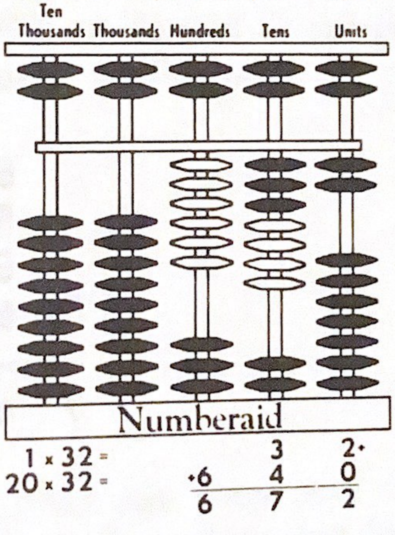
You can do this problem a third way (Figure 79).

(a) Multiply 1 x 32 = 32
(b) Multiply 20 x 32 = 640

(c) Add 672

This is the shortest way. To do it, you must remember how to multiply by 20 without hesitation.

Figure 79 (1 × 32) + (20 × 32) = 672



Now do the following multiplication problems, using the second method to check the first.

$$\begin{array}{rrrrr} 42 & 87 & 98 & 66 & 53 \\ \times\ 36 & 54 & 29 & 78 & 97 \\ \hline \end{array}$$

Multiplication by a Three-digit Multiplier

Problem: 569 (By addition) (Additive Method)

$$\begin{array}{r} 569 \\ \times\ 135 \\ \hline \end{array}$$

This problem says, take 569 five times; then add 5,690 three times; and finally add 56,900 once. On paper, it would look like this:

$$\begin{array}{r} 0\ 0\ 0\ 0\ 0 \\ 5\ 6\ 9 \\ 5\ 6\ 9 \\ 5\ 6\ 9 \\ 5\ 6\ 9 \\ 5\ 6\ 9 \\ 5\ 6\ 9\ 0 \\ 5\ 6\ 9\ 0 \\ 5\ 6\ 9\ 0 \\ 5\ 6\ 9\ 0\ 0 \\ \hline 7\ 6,\ 8\ 1\ 5 \end{array}$$

Problem:

$$\begin{array}{r} 569 \\ \times\ 135 \\ \hline \end{array}$$

This problem can be worked the pencil and paper way. This time the problem says:

$$\begin{array}{rlrrl}
 & (\ \ \ 9 \times \ \ \ 5 & &) = & 45 \\
45 + & (\ 60 \times \ \ \ 5 = & 300 &) = & 345 \\
345 + & (500 \times \ \ \ 5 = & 2{,}500 &) = & 2{,}845 \\
2{,}845 + & (\ \ \ 9 \times \ 30 = & 270 &) = & 3{,}115 \\
3{,}115 + & (\ 60 \times \ 30 = & 1{,}800 &) = & 4{,}915 \\
4{,}915 + & (500 \times \ 30 = & 15{,}000 &) = & 19{,}915 \\
19{,}915 + & (\ \ \ 9 \times 100 = & 900 &) = & 20{,}815 \\
20{,}815 + & (\ 60 \times 100 = & 6{,}000 &) = & 26{,}815 \\
26{,}815 + & (500 \times 100 = & 50{,}000 &) = & 76{,}815 \text{ Product}
\end{array}$$

This problem can be worked the short pencil and paper way also:

$$\begin{array}{rlrrl}
 & (569 \times \ \ \ 5 & &) = & 2{,}845 \\
2{,}845 + & (569 \times \ 30 = & 17{,}070 &) = & 19{,}915 \\
19{,}915 + & (569 \times 100 = & 56{,}900 &) = & 76{,}815 \text{ Product}
\end{array}$$

The last way is usually written on paper as follows:

$$\begin{array}{r} 569 \\ \times\ 135 \\ \hline 2845 \\ 17070 \\ 56900 \\ \hline 76{,}815 \end{array}$$

It is the most difficult, and requires that you know and remember all you have learned in the second way. It is really a short cut for the second way.

Do the following problems by at least two methods:

11 x 11	12 22	13 33	14 44	15 55	16 66	17 77	18 88	19 99
11 99	21 88	32 77	43 66	54 55	65 44	76 33	87 22	98 11
19 98	28 76	37 54	46 32	55 11	64 21	73 43	82 65	91 87
199 19	278 28	357 37	436 46	515 55	624 64	743 73	862 82	981 91
30 4	26 53	27 7	45 3	806 6	764 86	540 9	932 92	6,073 8
981 101	908 102	208 209	109 101	938 6	83 16	145 8	54 47	35 5
19 73	70 564	267 93	90 470	8,005 6	760 100	206 43	850 78	340 63
3.69 25		27.5 13		76 468	458 234		1.05 8	

SUBTRACTION

Problem: 1 (1 — 1 = 0) or (one minus one equals zero).

— 1

——

0

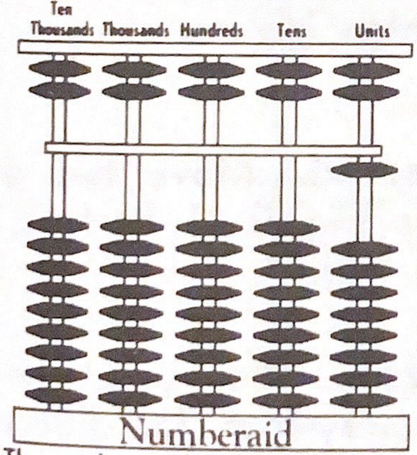
The problem asks, how can one (1) be divided into two smaller quantities, one of them being one unit (1). The answer, obviously, is that it cannot be divided, since it is one unit. There is no difference between the minuend and the subtrahend. No difference can be calculated. No difference is written as zero. This can be demonstrated on the Numberaid as follows.

Clear your Numberaid. Move up one green unit-counter against the stop. (Figure 80, Part A). Take away one green unit-counter from the stop.

The difference is zero, for there are no counters against the stop (Figure 80, Part B).

Figure 80 1-1=0

Part A 1-1



The number subtracted is an outlined counter.

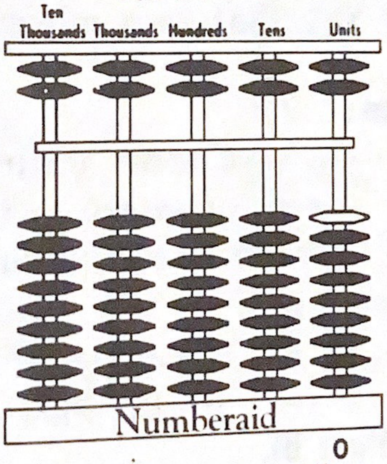
1

-1

One, take away one.

Figure 80 1-1=0

Part B 1-1=0



is

0

Zero

Work the following problems on your Numberaid.

$$\begin{array}{rrrrrrrrrr} 2 & 3 & 4 & 5 & 6 & 10 & 100 & 1000 & 13 & 56\\ -2 & 3 & 4 & 5 & 6 & 10 & 100 & 1000 & 13 & 56\\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

You can see that any number, minus the same number, equals zero (any number — the same number = 0)

$$\left(\begin{array}{r}\text{any number}\\ -\text{the same number}\\ \hline \text{Zero}\end{array}\right)$$

You can see there is no (zero) difference between two equal numbers.

Problem:
$$\begin{array}{r} 0\\ -1\\ \hline 0 \end{array}$$

Clear your Numberaid. Can you take away any counters? Naturally not, since there are none against the stop. Each of the following problems can be solved in the same way.

$$\begin{array}{rrrr} 0 & 0 & 0 & 0\\ -2 & 4 & 36 & 768\\ \hline & & & \end{array}$$

You can say that zero minus any number is zero.

Problem:
$$\begin{array}{r} 2\\ -1\\ \hline \end{array}$$

Clear your Numberaid. Move two green unit-counters up to the stop (Figure 81, Part A). Take away one. The difference is one (Figure 81, Part B).

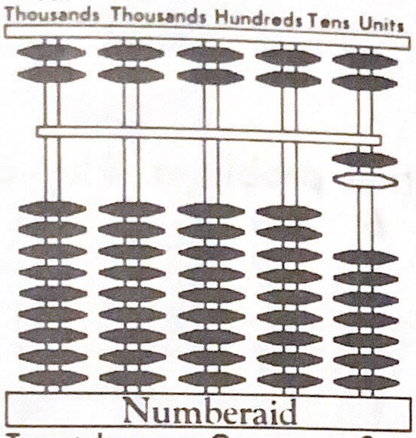
Problem:
$$\begin{array}{r} 6\\ -4\\ \hline \end{array}$$

Clear your Numberaid. Show six at the stop (Figure 82, Part A). Take away four. The difference is two (Figure 82, Part B).

Figure 81

2 - 1 = 1

Part A 2 - 1

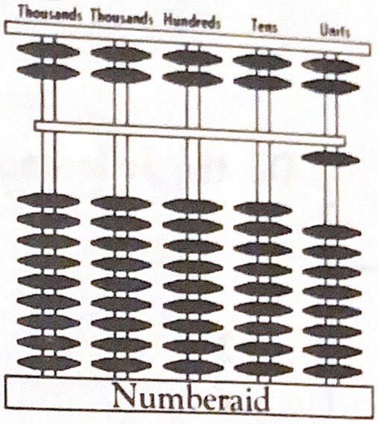


Two, take away One. 2
-1

Figure 81

2 - 1 - 1

Part B 2 - 1 = 1

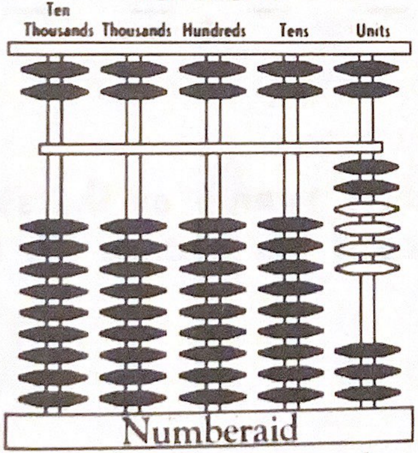


is 1
One

Figure 82

6 - 4 = 2

Part A 6 - 4

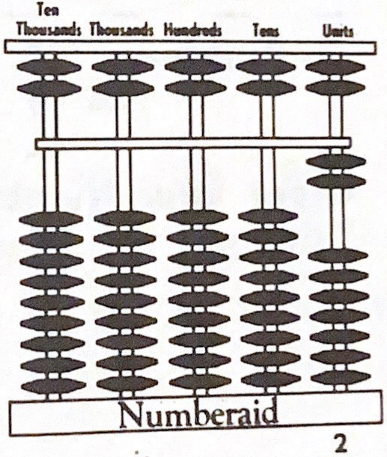


Six, take away Four. 6
-4

Figure 82

6 - 4 - 2

Part B - 2



is 2
Two

Do the following subtraction problems. Play Games 23 and 24.

$$\begin{array}{rrrrcrrrrrrrr}
2 &&&&\qquad& 6 & 6 & 6 & 6 & 6 &&& \\
\underline{-1} &&&&& \underline{1} & \underline{2} & \underline{3} & \underline{4} & \underline{5} &&& \\
3 & 3 &&&& 7 & 7 & 7 & 7 & 7 & 7 && \\
\underline{1} & \underline{2} &&&& \underline{1} & \underline{2} & \underline{3} & \underline{4} & \underline{5} & \underline{6} && \\
4 & 4 & 4 &&& 8 & 8 & 8 & 8 & 8 & 8 & 8 & \\
\underline{1} & \underline{2} & \underline{3} &&& \underline{1} & \underline{2} & \underline{3} & \underline{4} & \underline{5} & \underline{6} & \underline{7} & \\
5 & 5 & 5 & 5 && 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 \\
\underline{1} & \underline{2} & \underline{3} & \underline{4} && \underline{1} & \underline{2} & \underline{3} & \underline{4} & \underline{5} & \underline{6} & \underline{7} & \underline{8}
\end{array}$$

Problem: $\begin{array}{r} 20 \\ \underline{-\ 10} \end{array}$

Clear your Numberaid. Show twenty at the stop (Figure 83, Part A). Take away ten. The difference is ten (Figure 83, Part B).

Figure 83 20-10-10
Part A 20-10

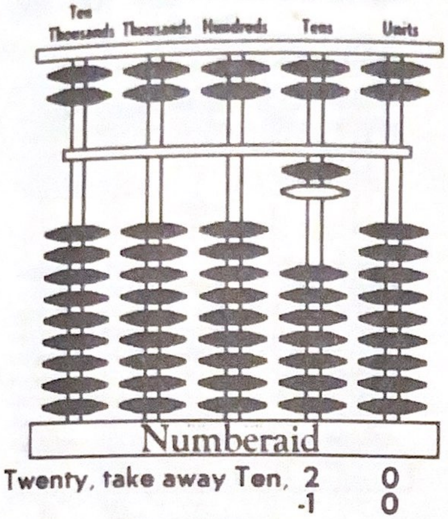
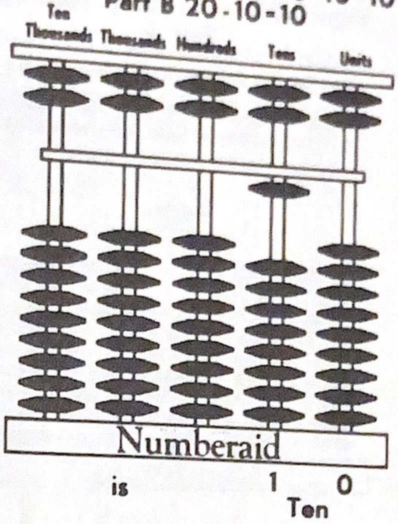


Figure 83 20-10-10
Part B 20-10-10



Do the following subtraction problems:

20				60	60	60	60	60			
−10				10	20	30	40	50			
—				—	—	—	—	—			
30	30			70	70	70	70	70	70		
10	20			10	20	30	40	50	60		
—	—			—	—	—	—	—	—		
40	40	40		80	80	80	80	80	80	80	
10	20	30		10	20	30	40	50	60	70	
—	—	—		—	—	—	—	—	—	—	
50	50	50	50	90	90	90	90	90	90	90	90
10	20	30	40	10	20	30	40	50	60	70	80
—	—	—	—	—	—	—	—	—	—	—	—

Figure 84 200 - 100 = 100
Part A

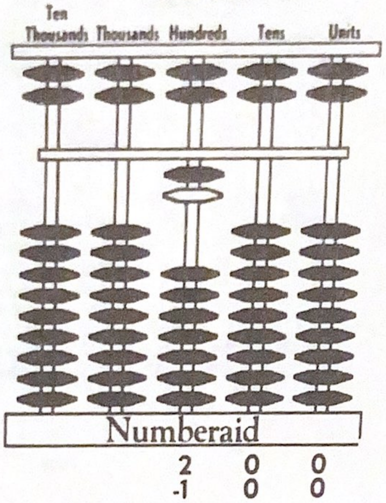
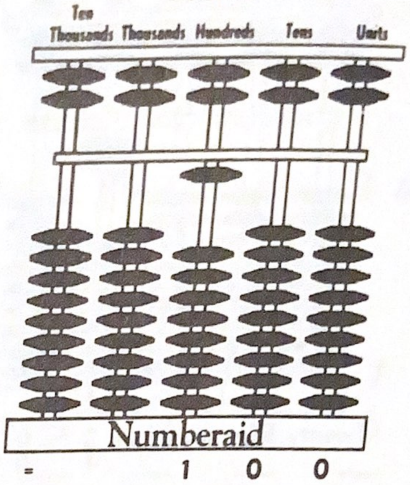


Figure 84 200 - 100 = 100
Part B



Problem: 200
— 100 **Clear your Numberaid.** Show 200 at the stop
—— (Figure 84, Part A). Take away 100. The difference
is 100 (Figure 84, Part B).

Problem: 3000
— 2000 **Clear your Numberaid.** Show 3.000 at the stop
—— (Figure 85, Part A). Take away 2,000. The difference is 1,000 (Figure 85, Part B).

Problem: 40,000
— 30,000 **Clear your Numberaid.** Show 40,000 at the
—— stop (Figure 86, Part A). Take away 30,000.
The difference is 10,000 (Figure 86, Part B).

Figure 85 3000 - 2000 = 1000
Part A

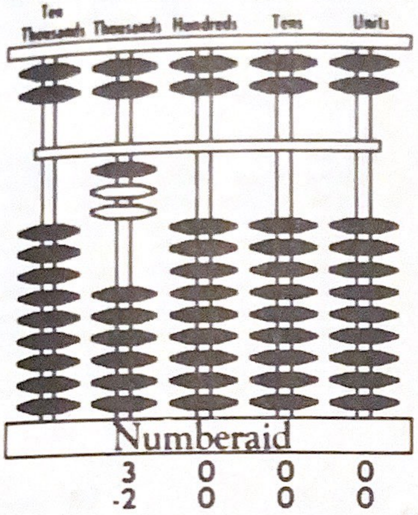


Figure 85 3000 - 2000 = 1000
Part B

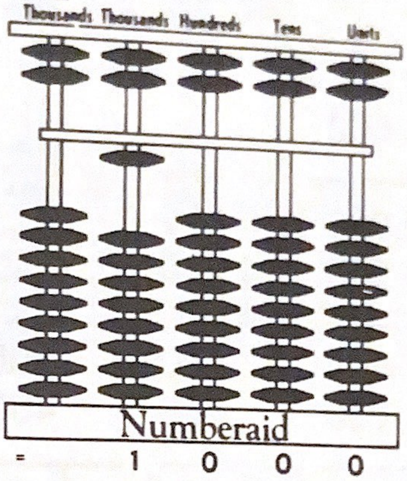


Figure 86
40,000 - 30,000 = 10,000
Part A

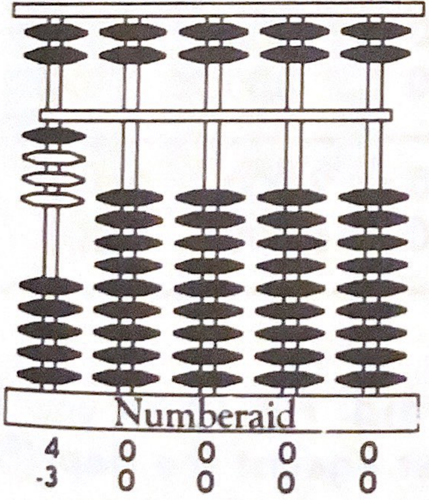
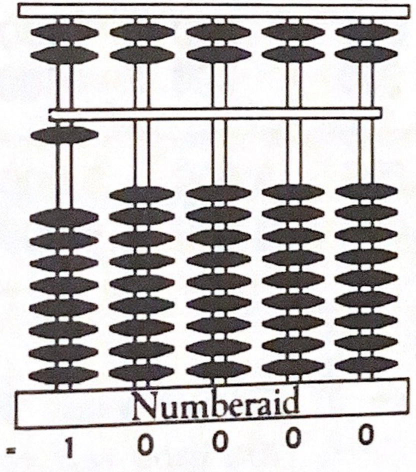


Figure 86
40,000 - 30,000 = 10,000
Part B



Do the following subtraction problems:

200
— 100

3,000	3,000
1,000	2,000

40,000	40,000	40.000
10,000	20,000	30,000

500	500	500	500
100	200	300	400

6,000	6,000	6,000	6,000	6,000
1,000	2,000	3,000	4,000	5,000

70,000	70,000	70,000	70,000	70,000	70,000
10,000	20,000	30,000	40,000	50,000	60,000

800	800	800	800	800	800	800
100	200	300	400	500	600	700

9,000	9,000	9,000	9,000	9,000	9,000	9,000	9,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000

Problem: 10
— 5
—

Clear your Numberaid. Put 10 on your Numberaid (one red ten-counter against the stop) (Figure 87, Part A).

Figure 87 10 - 5 = 5

Part A

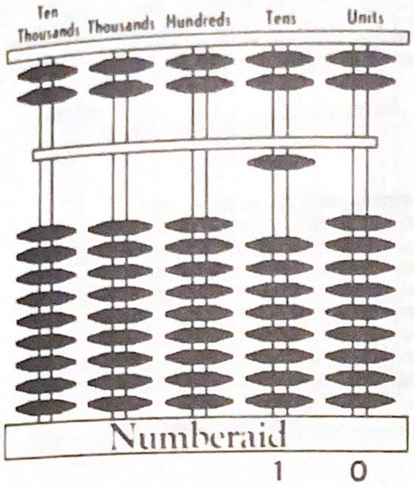


Figure 87 10 - 5 = 5

Part B

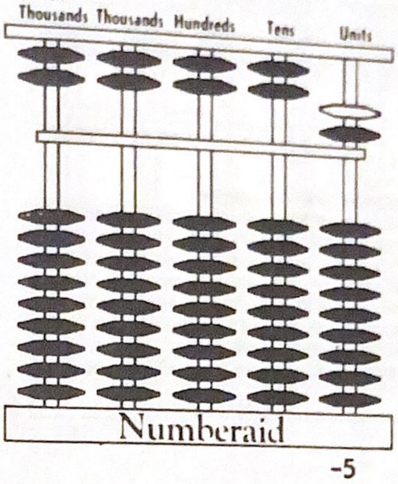
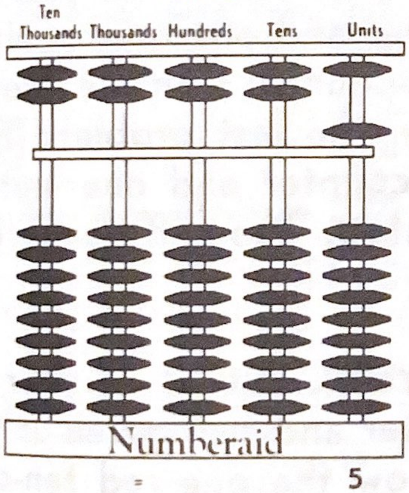


Figure 87 10 - 5 = 5

Part C



Borrow the 10 to make two five-unit-counters in the units column. They are pushed down to the stop as the red ten-counter is pushed away from the stop. This is called "borrowing one ten" (Figure 87, Part B). Take away five by pushing one five-unit-counter up from the stop. One five-unit-counter is still at the stop. The difference is five (5) (Figure 87, Part C).

Figure 88 11 - 6 = 5

Part A

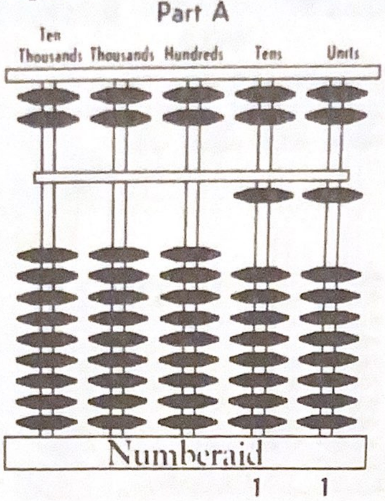
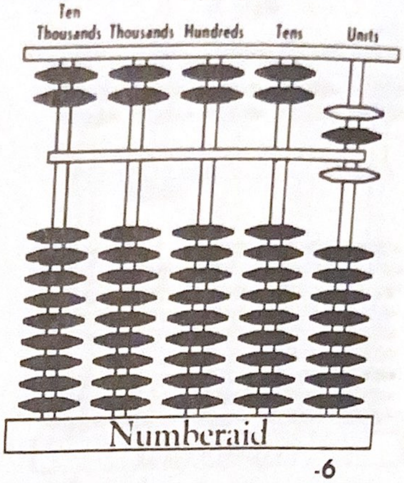


Figure 88 11 - 6 = 5

Part B



Problem: 11
— 6 **Clear your Numberaid.** Put 11 on your Number-
— aid (one red ten-counter and one green unit-counter)

(Figure 88, Part A). Borrow the one red ten-counter to make two five-unit-counters in the units column, as you did in the last problem (Figure 88, Part B). Take away six units (one five-unit-counter and one unit-counter). One five-unit-counter is still against the stop. The difference is five (5) (Figure 88, Part C).

Problem: 12
— 8 **Clear your Numberaid.** Put 12 on your Number-
— aid (one red ten-counter and two green unit-counters

against the stop) (Figure 89, Part A). Borrow the one red ten-counter to make two five-unit-counters in the units column. You still cannot take away eight, for there is only one green unit-counter at the stop. So you must change one of the green five-unit-counters above the stop for five green unit-counters below the stop.

Figure 88 11 - 6 = 5

Part C

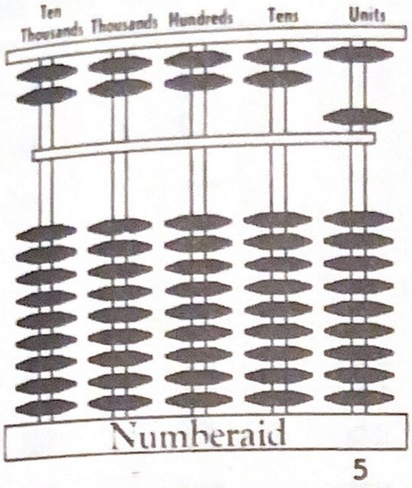


Figure 89

Part A 12 - 8 = 4

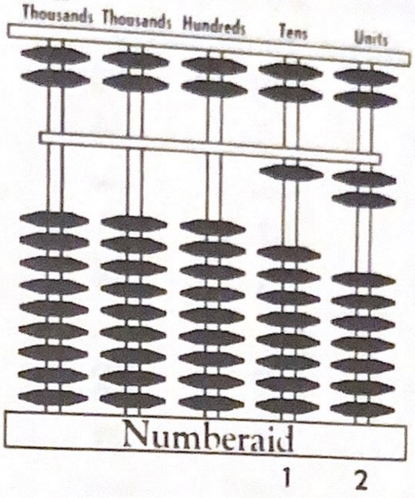


Figure 89 12 - 8 = 4

Part B

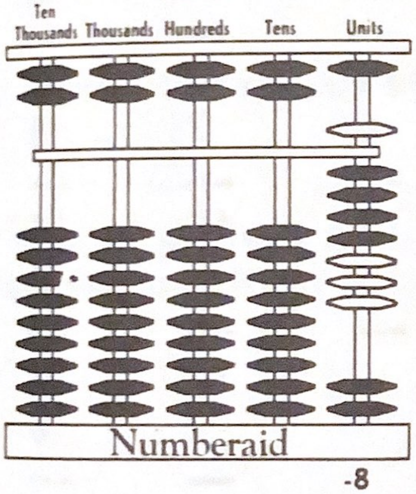
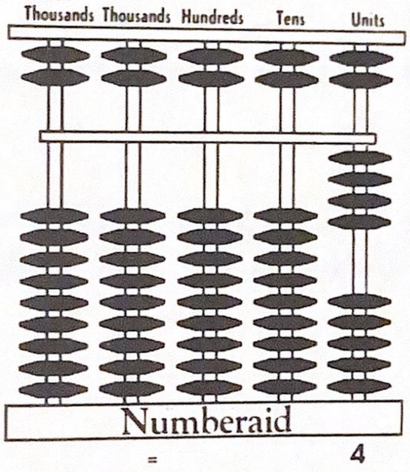


Figure 89

Part C 12 - 8 = 4



Then take away eight units (one green five-unit-counter and three green unit-counters) (Figure 89, Part B). Four green unit-counters are left at the stop. The difference is four (4) (Figure 89, Part C).

Do the following subtraction problems in which you have to "borrow" from tens place. Be sure to write out the problems and the differences as you have done previously.

10 — 1 —	10 2 —	10 3 —	10 4 —	10 5 —	10 6 —	10 7 —	10 8 —	10 9 —
	11 2 —	11 3 —	11 4 —	11 5 —	11 6 —	11 7 —	11 8 —	11 9 —
		12 3 —	12 4 —	12 5 —	12 6 —	12 7 —	12 8 —	12 9 —
			13 4 —	13 5 —	13 6 —	13 7 —	13 8 —	13 9 —
				14 5 —	14 6 —	14 7 —	14 8 —	14 9 —
					15 6 —	15 7 —	15 8 —	15 9 —
						16 7 —	16 8 —	16 9 —
							17 8 —	17 9 —
								18 9 —

$$\begin{array}{r} 20 \\ -\ 1 \\ \hline \\ \end{array} \quad \begin{array}{r} 20 \\ 2 \\ \hline \\ \end{array} \quad \begin{array}{r} 20 \\ 3 \\ \hline \\ \end{array} \quad \begin{array}{r} 20 \\ 4 \\ \hline \\ \end{array} \quad \begin{array}{r} 20 \\ 5 \\ \hline \\ \end{array} \quad \begin{array}{r} 20 \\ 6 \\ \hline \\ \end{array} \quad \begin{array}{r} 20 \\ 7 \\ \hline \\ \end{array} \quad \begin{array}{r} 20 \\ 8 \\ \hline \\ \end{array} \quad \begin{array}{r} 20 \\ 9 \\ \hline \\ \end{array}$$

$$\begin{array}{r} 90 \\ 1 \\ \hline \\ \end{array} \quad \begin{array}{r} 90 \\ 2 \\ \hline \\ \end{array} \quad \begin{array}{r} 90 \\ 3 \\ \hline \\ \end{array} \quad \begin{array}{r} 90 \\ 4 \\ \hline \\ \end{array} \quad \begin{array}{r} 90 \\ 5 \\ \hline \\ \end{array} \quad \begin{array}{r} 90 \\ 6 \\ \hline \\ \end{array} \quad \begin{array}{r} 90 \\ 7 \\ \hline \\ \end{array} \quad \begin{array}{r} 90 \\ 8 \\ \hline \\ \end{array} \quad \begin{array}{r} 90 \\ 9 \\ \hline \\ \end{array}$$

If you solve 10 and 20, what is the difference of 30? 40? 90?

$$\begin{array}{r} 10 \\ -\ 1 \\ \hline \\ \end{array} \quad \begin{array}{r} 20 \\ 1 \\ \hline \\ \end{array} \quad \begin{array}{r} 30 \\ 1 \\ \hline \\ \end{array} \quad \begin{array}{r} 40 \\ 1 \\ \hline \\ \end{array} \quad \begin{array}{r} 90 \\ 1 \\ \hline \\ \end{array}$$

What kind of pattern does the above table suggest? If you know the pattern can you use it to solve a problem like 98 quickly?

$$\begin{array}{r} 98 \\ -\ 9 \\ \hline \\ \end{array}$$

Problem: 100
— 10
____

Clear your Numberaid. Put 100 on it (Figure 90, Part A). Borrow one hundred, as one-fifty-counter and five-ten-counters in tens place (Figure 90, Part B). Take away one ten. The difference is ninety (Figure 90, Part C).

Do the following problems. The illustration will help to work them correctly.

100	100	100	100	100	100	100	100	100
— 10	20	30	40	50	60	70	80	90

1,100	1,100	1,100	1,100	1,100	1,100	1,100	1,100
200	300	400	500	600	700	800	900

12,000	12,000	12,000	12,000	12,000	12,000	12,000
3,000	4,000	5,000	6,000	7,000	8,000	9,000

230	230	230	230	230	230
40	50	60	70	80	90

3,400	3,400	3,400	3,400	3,400
500	600	700	800	900

Figure 90 Part A 100 - 10 = 90

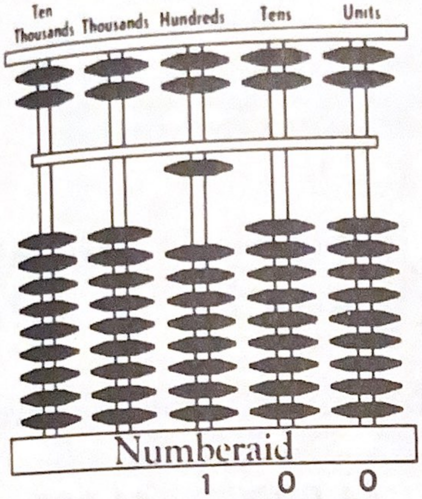


Figure 90 Part B 100 - 10 = 90

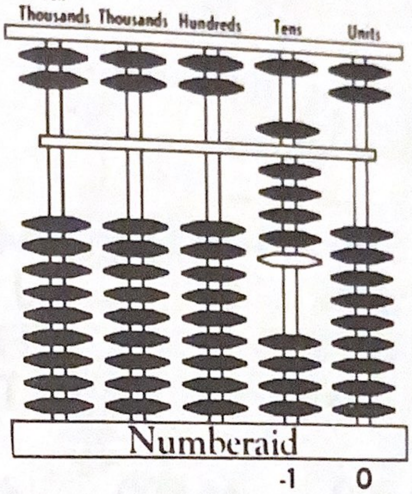
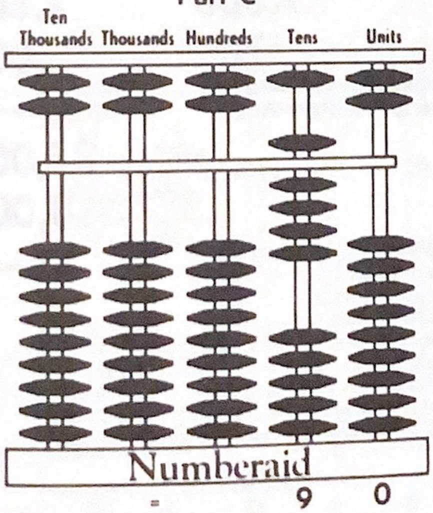


Figure 90 Part C 100 - 10 = 90



Problem: 44,000
— 6,000

Clear your Numberaid. Put 44,000 on it (Figure 91, Part A). Borrow ten thousand as two five thousand counters (Figure 91, Part B). Take away 6,000. The difference is 38,000 (Figure 91, Part C).

Do the following problems. The illustration will help to work them correctly.

44,000	44,000	44,000	44,000	44,000
— 5,000	6,000	7,000	8,000	9,000
	650	650	650	650
	60	70	80	90
		7,600	7,600	7,600
		700	800	900
			87,000	87,000
			8,000	9,000
				980
				90

Figure 91

44,000 - 6,000 = 38,000

Part A

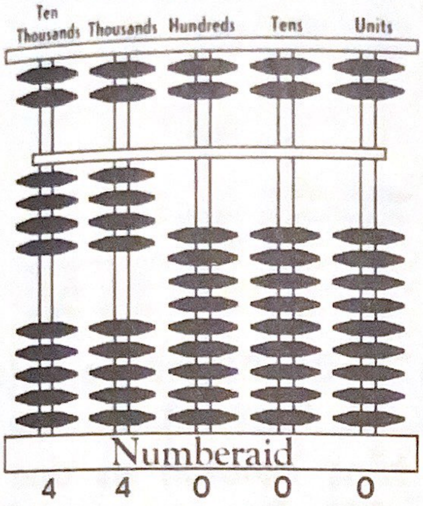


Figure 91

44,000 - 6,000 = 38,000

Part B

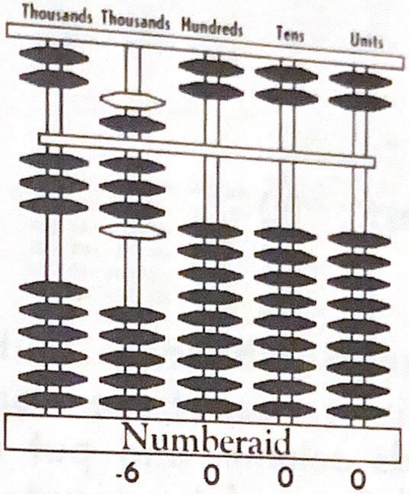
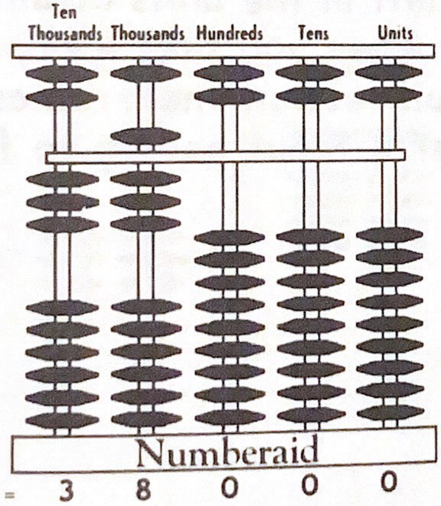


Figure 91

44,000 - 6,000 = 38,000

Part C



Problem: 100
— 1

Clear your Numberaid. Put 100 on your Numberaid (one white hundred-counter against the stop) (Figure 92, Part A). Borrow 100 from the hundreds column, and put it in the tens column as one red fifty-counter and five red ten-counters (Figure 92). Borrow one 10 from the tens column, and put it in the units column as one green five-counter and five green unit-counters. Take away one green unit-counter (Figure 92, Part B). One red fifty-counter and four red ten-counters, or nine tens, are left in the tens column. One green five-counter and four green unit-counters, or nine units, are left in the units column. The number remaining is 99, and is the remainder when you take away one from 100 (Figure 92, Part C). Having done this subtraction once, remember that 100 — 1 = 99, so you can write the answer without having to figure it out.

Figure 92 100 - 1 = 99

Part A

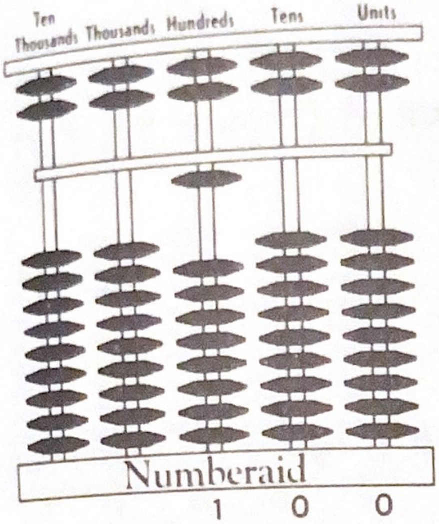


Figure 92 100 - 1 = 99

Part B

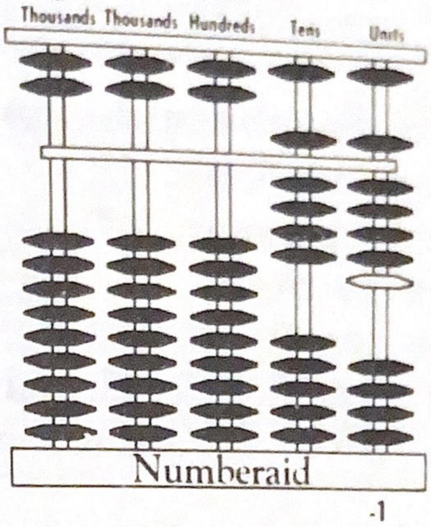
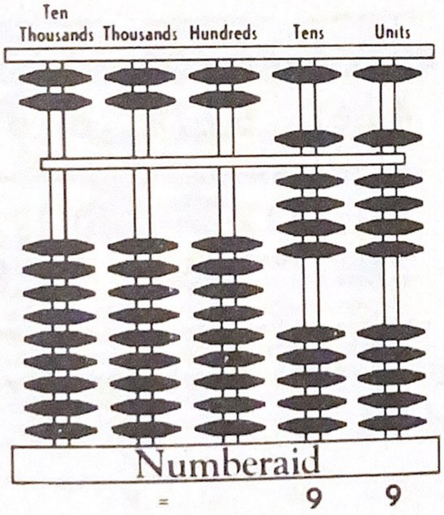


Figure 92 100 - 1 = 99

Part C



Problem: 10,000
— 1,234

Clear your Numberaid. Put 10,000 on it (Figure 93, Part A). Borrow ten thousand as follows:

Nine thousands
Nine hundreds
Nine tens
Ten units (Figure 93, Part B).

Take away 1,234. The difference is 8,766 (Figure 93, Part C).

Do the following subtraction problems:

200	300	400	500	600	700	800	900	
— 2	3	4	5	6	7	8	9	
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
12	23	34	45	56	67	78	89	90
10,000	20,000	30,000	40,000	50.000	60,000	70,000	80,000	90,000
1,234	2,345	3,456	4,567	5,678	6,789	7,890	8,901	9,012
203	1,020	1,234	3,451	12,345	30,321	34,567		
26	936	987	2,862	3,456	5,678	23,678		

Now you can do any subtraction problem involving any whole numbers. Play Games 21, 22, 23 and 24.

Figure 93

10 000 - 1 234 = 8 766

Part A

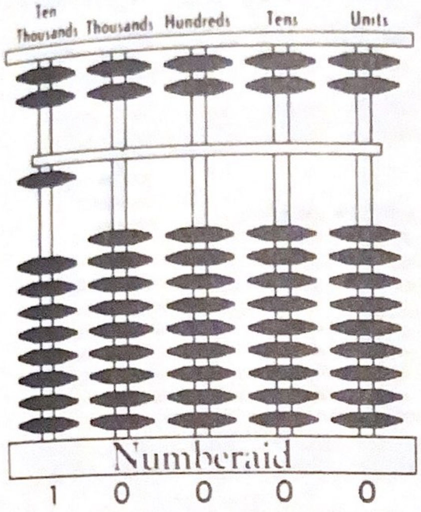


Figure 93

10 000 - 1 234 = 8 766

Part B

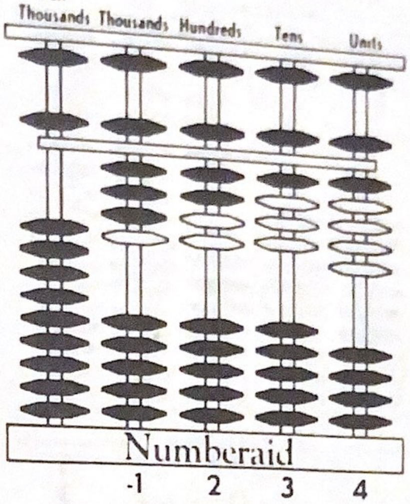


Figure 93

10,000 - 1,234 = 8,766

Part C

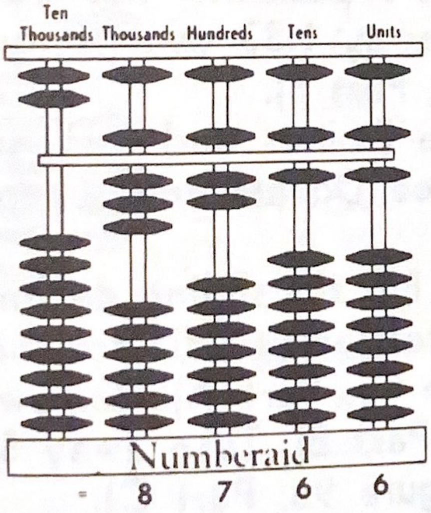


Figure 94 $2.75 - $1.33 = $1.42
Part A

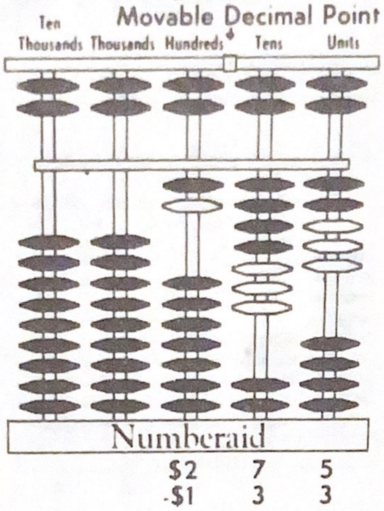
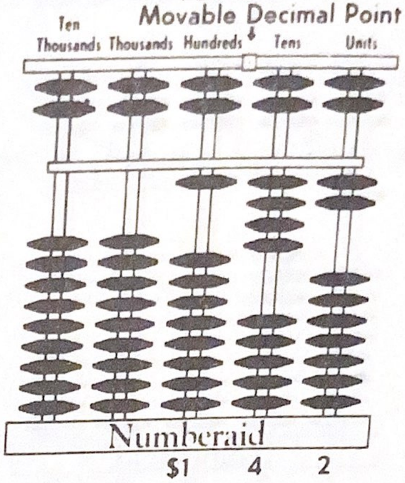


Figure 94 $2.75 - $1.33 = $1.42
Part B



Problem: $2.75 Two Dollars and seventy-five cents
— 1.33 One Dollar and thirty-three cents

Clear your Numberaid. Put the sliding decimal point between the red and white columns of counters. The white counters will then be dollars, the red and green counters pennies. Put $2.75 on your Numberaid (Figure 94, Part A). Take away 1.33 cents (Figure 94, Part A). The difference is $1.42 (Figure 94, Part B).

Problem: $5.46 Five Dollars and forty-six cents
— 3.78 Three Dollars and seventy-eight cents

Clear your Numberaid. Put the sliding decimal point between the red and white columns of counters as you did for the last problem. Put $5.46 on your Numberaid (Figure 95, Part A). Borrow from the tens and hundreds columns (Figure 95, Part B). Take away $3.78 (Figure 95, Part B). The difference is $1.68 (Figure 95, Part C).

Figure 95 $5.46 - $3.78 = $1.68
Part A

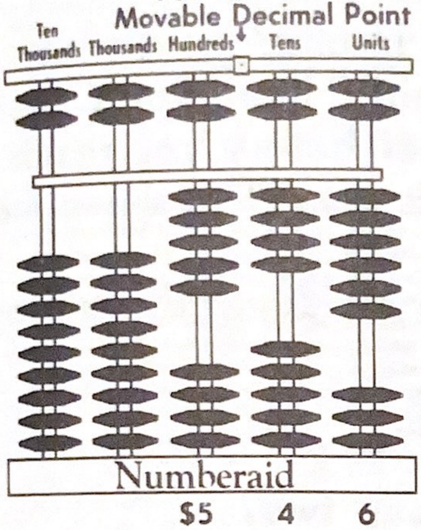


Figure 95 $5.46 - $3.78 = $1.68
Part B

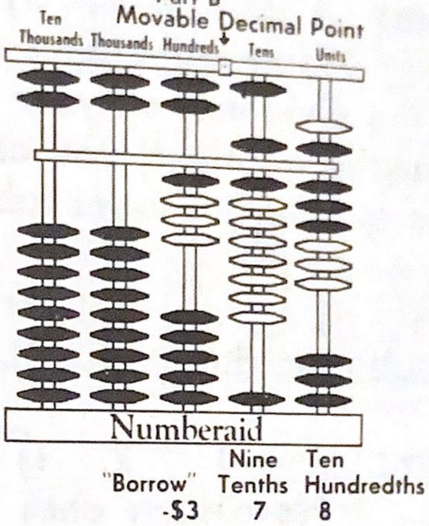
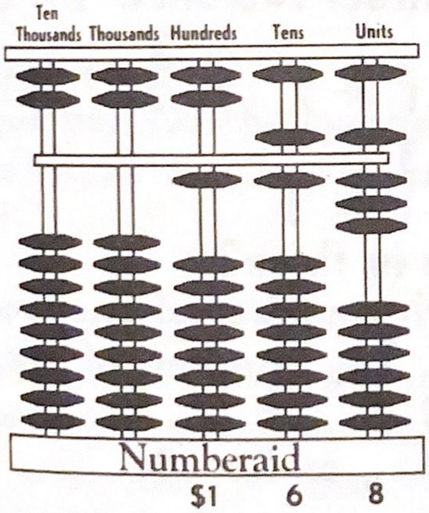


Figure 95 $5.46 - $3.78 = $1.68
Part C



Now do the following subtraction problems:

$1.55 — .36	$3.87 1.43	$12.98 6.75
$1 08 1.03	$ .98 .48	$ .08 .04
$10.50 8.20	$3.45 1.89	$5.06 3.78
$27.52 13.66	$305.78 177.98	

DIVISION

Problem: $1 \div 1 = 1$, $\begin{array}{r}1\\1\overline{)1}\end{array}$, $\frac{1}{1} = 1$

How many ones are there in one?

Show one (the dividend) on your Numberaid (Figure 96). Count the number of ones (the quotient) you can subtract before you reach zero. The number (the quotient) of ones subtracted is one. This shows that:

$1 \div 1 = 1$, $\begin{array}{r}1\\1\overline{)1}\end{array}$, $\frac{1}{1} = 1$, or one (the dividend) divided by one (the divisor) equals one (the quotient).

Problem: $2 \div 1 = 2$, $\begin{array}{r}2\\1\overline{)2}\end{array}$, or $\frac{2}{1} = 2$

How many ones are there in two?

Put two on your Numberaid (Figure 97). Count the number of ones you can subtract to get zero. You can subtract two ones. The quotient of this division is two. This shows that: $2 \div 1 = 2$, $\begin{array}{r}2\\1\overline{)2}\end{array}$, $\frac{2}{1} = 2$.

Problem: $3 \div 1 = 3$, $\begin{array}{r}3\\1\overline{)3}\end{array}$, or $\frac{3}{1} = 3$

How many ones are there in three?

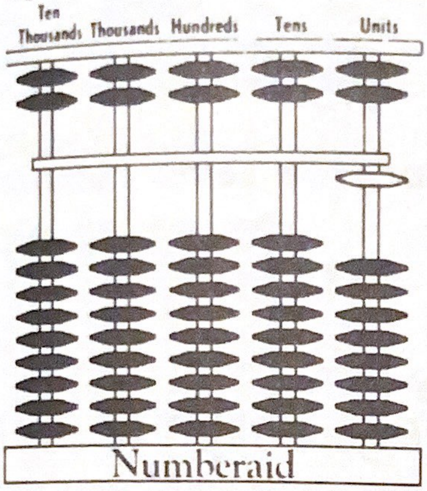
Put three on your Numberaid. Count the number of ones you can subtract to get zero. You can subtract three. The quotient of this division is three. This shows that: $3 \div 1 = 3$, $\begin{array}{r}3\\1\overline{)3}\end{array}$, or $\frac{3}{1} = 3$

Problem: $12 \div 1$, $1\overline{)12}$, $\frac{12}{1}$

How many ones are there in 12?

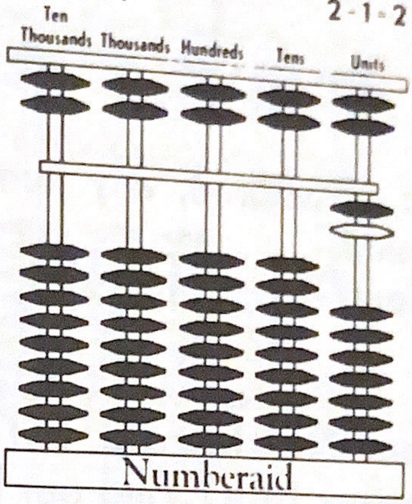
Put 12 on your Numberaid (Figure 98). How many ones (what number)

Figure 96 1 - 1 = 1



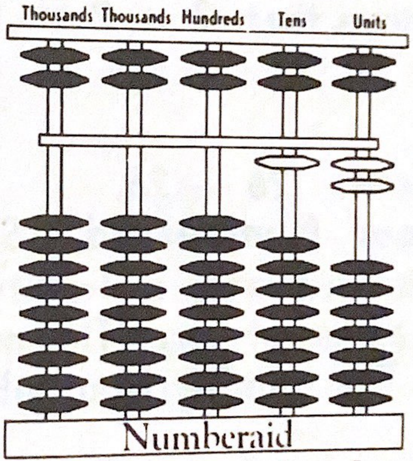
$$\frac{1}{1} = 1 \qquad 1\overset{1}{\overline{)1}} \qquad \begin{array}{r}1\\-1\\\hline 0\end{array} \text{ (One One - Quotient 1)}$$

Figure 97 2 - 1 = 2



$$\frac{2}{1} = 2 \text{ (Divisor)} \qquad 1\overset{2 \text{ (Quotient)}}{)\,2 \text{ (Dividend)}} \qquad \begin{array}{r}2\\-1\\-1\\\hline 0\end{array} \begin{array}{l}\text{(Dividend)}\\\text{(Two Ones -}\\\\\text{Quotient 2)}\end{array}$$

Figure 98 12 - 1 = 12



$$\frac{12}{1} = 12 \qquad 1\overset{12}{\overline{)12}}. \qquad \begin{array}{r}1\\-1\\\hline\end{array} \quad \begin{array}{r}2\\2\\\hline 0\end{array} \text{ (Quotient 12)}$$

can you subtract to get zero? The number 12. The quotient of this division is 12. This shows that:

$$12 \div 1 = 12,\ 1\overset{12}{\overline{)\,12}},\ \ \frac{12}{1} = 12$$

Can you say that a number, divided by one, equals the number (a number ÷ one = the number)? Why?

Do the following problems in division:

$4 \div 1$ $\quad \frac{7}{1} =$ $\quad 15 \div 1 =$ $\quad \frac{635}{1} =$

$1\overline{)6}$ $\quad 9 \div 1 =$ $\quad 1\overline{)21}$ $\quad 8956 \div 1 =$

Problem: Dividend, any number. Divisor, the same number. What is the quotient?

Problem: $2 \div 2$, $2\overline{)2}$, $\frac{2}{2}$

How many twos are there in two?

Clear your Numberaid. Put two on it (Figure 99). Two can be divided into how many twos? Find out by subtracting twos until you get zero. The number of twos you subtract is the quotient. The number subtracted is one (1), and is the quotient.

This shows that: $2 \div 2 = 1$, $\begin{array}{r}1\\2\overline{)2}\end{array}$, $\frac{2}{2} = 1$, or two divided by two equals one.

Problem: $26 \div 26$, $26\overline{)26}$, $\frac{26}{26}$

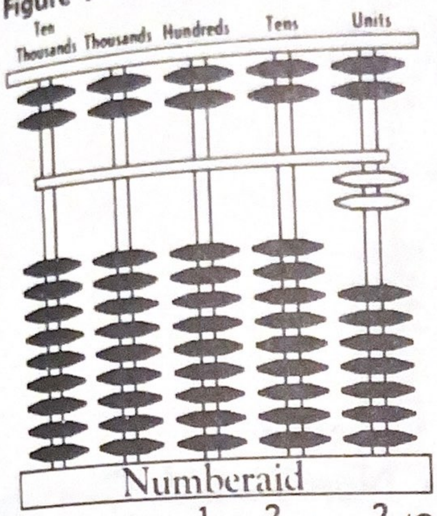
Clear your Numberaid. Show 26 on it (Figure 100). How many twenty-sixes are there in twenty-six? Find out by subtracting twenty-sixes until you arrive at zero. The number of twenty-sixes you subtract is the quotient. The number you subtracted is one (1), and is the quotient.

This shows that: $26 \div 26 = 1$, $\begin{array}{r}1\\26\overline{)26}\end{array}$, $\frac{26}{26} = 1$

Now do the following division problems. If you do each problem correctly, and remember the quotient, you will never have to work it out again. You can write it immediately. You must remember how to do the

Figure 99

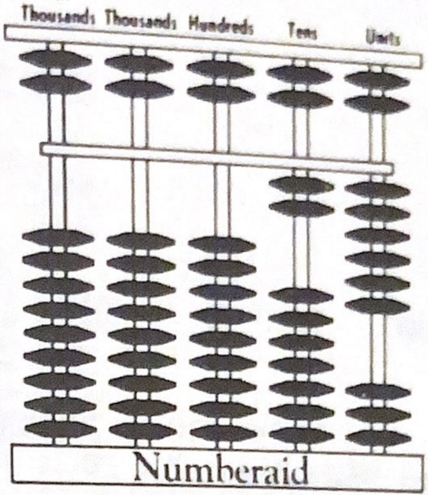
$2 \div 2 = 1$



$\begin{array}{r}1\\2\overline{)2}\end{array}$ $\quad \frac{2}{2}=1 \quad$ $\begin{array}{r}2\\-2\\\hline 0\end{array}$ (One Two - Quotient 1)

Figure 100

$26 \div 26 = 1$



$\begin{array}{r}1\\26\overline{)26}\end{array}$ $\quad \frac{26}{26}=1 \quad$ $\begin{array}{r}2\\-2\end{array}$ $\begin{array}{r}6\\6\\\hline 0\end{array}$ (One Twenty-Six - Quotient 1)

division, however, so that if you forget the quotient you can work it out again.

$5 \div 5, \qquad 5\overline{)\,5} \;, \qquad \frac{5}{5}$

$365 \div 365, \qquad 365\overline{)\,365} \;, \qquad \frac{365}{365}$

$2687 \div 2687 \qquad 2687\overline{)\,2687} \qquad \frac{2687}{2687}$

$33{,}678 \div 33{,}678, \qquad 33{,}678\overline{)\,33{,}678} \qquad \frac{33{,}678}{33{,}678}$

You can see that a number, divided by the same number, equals one (a number ÷ the same number = 1).

Problem: $4 \div 0$, $0\overline{)\,4}$, $\frac{4}{0}$

How many zeros are there in four?

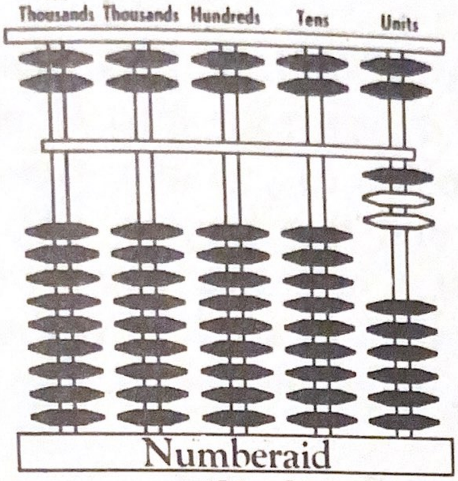
Clear your Numberaid. Put four on your Numberaid. Can you divide by zero? Of course not. The problem is impractical. Any number divided by zero is zero.

Problem: $0 \div 3$, $3\overline{)\,0}$, $\frac{0}{3}$

How many threes are there in zero?

Put zero on your Numberaid. Can you divide by three? Of course not. You still have zero. Is this true any time you try to divide zero by a number? Does zero divided by any number equal zero ($0 \div$ any number $= 0$)?

Figure 101 $3 \div 2 = 1\frac{1}{2}$



$\begin{array}{r} 1\text{R}1 \\ 2\overline{)3} \end{array}$ $\frac{3}{2} = 1\frac{1}{2}$ $\begin{array}{r} 3 \\ -2 \\ \hline 1 \end{array}$ Remainder

Problem: $3 \div 2$, $2\overline{)\,3}$, $\frac{3}{2}$

How many twos are there in three?

Clear your Numberaid. Put three (the dividend) on your Numberaid (Figure 101). To divide three into as many twos as you can, take away two (the divisor) as often as you can. Take away one two. Can you take away another two? Of course not. Why not? You can divide three into one two, and one one.

The quotient is one, which is the number of times you were able to take two (the divisor) away from three (the dividend). The one that was left at the stop is called the "Remainder." It is written after the quotient as R1, meaning "Remainder of 1." Or the quotient could be written as $1\frac{1}{2}$, with the remainder of 1 as the numerator of the fraction, and the divisor of 2 as the denominator.

$3 \div 2 = 1\frac{1}{2}$, $\begin{array}{r} 1\text{ R}1 \\ 2\overline{)\,3} \end{array}$, $\frac{3}{2} = 1\frac{1}{2}$

Problem: $4 \div 2$, $2\overline{)4}$, $\frac{4}{2}$

Clear your Numberaid. Put four (the dividend) on your Numberaid (Figure 102, Part A). To divide four into as many twos as you can, take away two (the divisor) as often as you can. Take away one two (Figure 102, Part A). Take away a second two (Figure 102, Part B). You have divided the four green unit-counters into two equal parts, each made up of two green unit-counters. No counters are left at the stop after you have taken away the second two. The quotient is two. The remainder is zero, or there is no remainder.

$$4 \div 2 = 2, \quad \begin{array}{r} 2 \\ 2\overline{)4} \end{array}, \quad \frac{4}{2} = 2$$

Problem: $5 \div 2$, $2\overline{)5}$, $\frac{5}{2}$

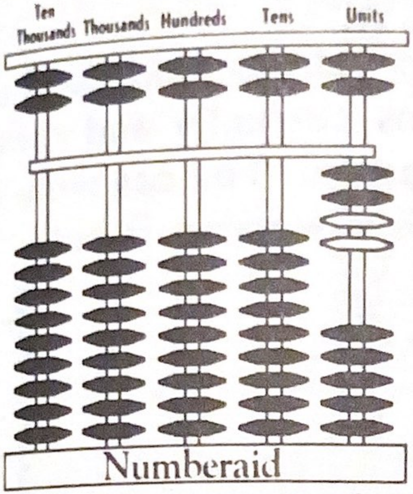
How many twos are there in five?

Clear your Numberaid. Put five (the dividend) on your Numberaid (Figure 103, Part A). To divide five into as many twos as you can, take away two (the divisor) as often as you can. Each time you move down two green unit-counters, you are taking two away from five. Take away the first two. There are three left (Figure 103, Parts A and B). Take away the second two. There is one left (Figure 103, Part B). You took two away two times, so the quotient is two. You cannot take two away again, since there is only one left, so the remainder is one. It is written after the quotient as R1, meaning "Remainder of 1" (quotient = 2 R1). Or the quotient could be written as $2\frac{1}{2}$, with the remainder of 1 as the numerator of the fraction, and the divisior of 2 as the denominator.

$$5 \div 2 = 2\tfrac{1}{2}. \quad \begin{array}{r} 2\text{ R}1 \\ 2\overline{)5} \end{array} \quad \frac{5}{2} = 2\tfrac{1}{2}$$

Figure 102 4 ÷ 2 = 2

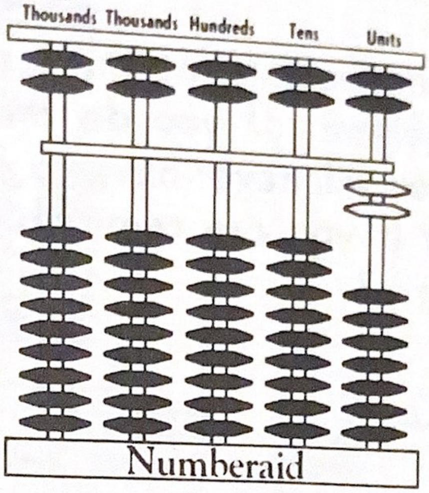
Part A



$$\begin{array}{r} 4 \\ -2 \end{array}$$ One Two

Figure 102 4 ÷ 2 = 2

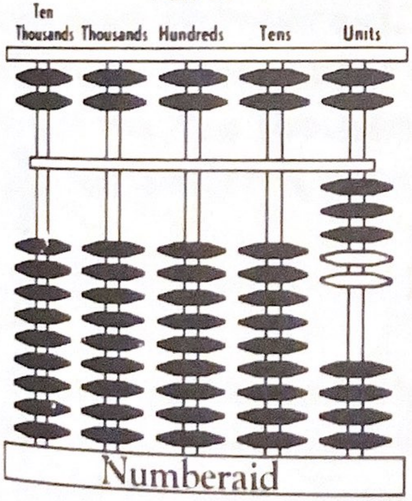
Part B



$2\overline{)4}^{\,2}$ $\frac{4}{2} = 2$ $\begin{array}{r} 2 \\ -2 \\ \hline 0 \end{array}$ (Two Two's - Quotient 2)

Figure 103 5 ÷ 2 = 2½

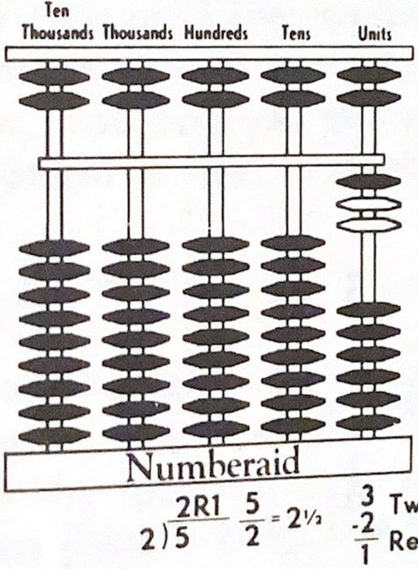
Part A



$$\begin{array}{r} 5 \\ -2 \end{array}$$ One Two

Figure 103 5 ÷ 2 = 2½

Part B



$2\overline{)5}^{\,2R1}$ $\frac{5}{2} = 2\frac{1}{2}$ $\begin{array}{r} 3 \\ -2 \\ \hline 1 \end{array}$ Two Twos Remainder

Follow the pattern of the last three problems, and do the followin division problems. If you do the problems correctly and remember th quotients, you will never have to do them again. You can write the answ immediately if you can remember it without working it out.

$6 \div 2$

$9 \div 2$

$6 \div 3$

$9 \div 3$

$7 \div 4$

$6 \div 5$

$9 \div 5$

$9 \div 6$

$9 \div 8$

$2\overline{)\ 7}$

$3\overline{)\ 4}$

$3\overline{)\ 7}$

$4\overline{)\ 5}$

$4\overline{)\ 8}$

$5\overline{)\ 7}$

$6\overline{)\ 7}$

$7\overline{)\ 8}$

$\frac{8}{2}$

$\frac{5}{3}$

$\frac{8}{3}$

$\frac{6}{4}$

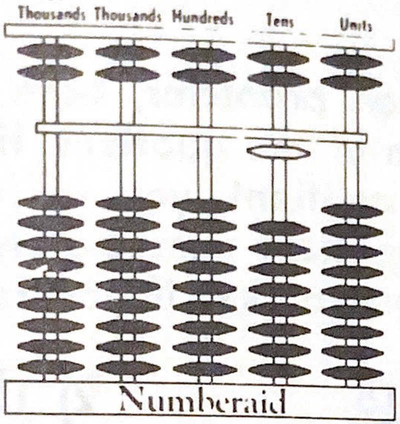
$\frac{9}{4}$

$\frac{8}{5}$

$\frac{8}{6}$

$\frac{9}{7}$

Figure 104 10 ÷ 2 = 5



$\begin{array}{r}5\\2\overline{)10}\end{array}$ $\frac{10}{2} = 5$ $\begin{array}{r}1\\-1\\\hline\text{Zero}\end{array}$ $\begin{array}{l}0 \text{ (Dividend)}\\0 \text{ (5 Quotient × 2 - Dividend)}\\\text{Check - Quotient correct}\end{array}$

Problem: $10 \div 2$, $2\overline{)\,10}$ $\frac{10}{2}$

How many twos are there in ten?

Put ten on your Numberaid. Subtract twos until the remainder is zero, or is less than two. Keep count of the number of twos you subtract on the blue counters in the ten thousands column. You find you can subtract five twos, and the remainder is zero. The quotient is five. You then can say:

$$10 \div 2 = 5 \qquad \begin{array}{r}5\\2\overline{)\,10}\end{array} \qquad \frac{10}{2} = 5$$

This process is rather lengthy. You can shorten it if you can mentally calculate that if you subtracted five twos, or ten, from the dividend, the remainder would be zero. Then you know the quotient is five. Put ten on your Numberaid (Figure 104). Subtract ten (Figure 104). The remainder is zero. The quotient is five. There is no remainder.

Do the following division problems, both by subtracting the divisor and by mental computation of the quotient. If you do each problem correctly and remember the quotient, you will never have to work it out again. You can write the quotient immediately. You must remember how to do the division, however, so that if you forget the quotient you can work it out again.

$2\overline{)\,12}$ $\quad 13 \div 2$ $\quad \frac{15}{2}$

$3\overline{)\,23}$ $\quad \frac{26}{3}$ $\quad 11 \div 4$

$4\overline{)\,32}$ $\quad \frac{35}{4}$ $\quad 12 \div 5$

$5\overline{)\,33}$ $\quad \frac{36}{5}$ $\quad 39 \div 5$

$6\overline{)\,29}$ $\quad \frac{33}{6}$ $\quad 37 \div 6$

$7\overline{)\,19}$ $\quad \frac{24}{7}$ $\quad 30 \div 7$

$8\overline{)\,11}$ $\quad \frac{16}{8}$ $\quad 22 \div 8$

$8\overline{)\,60}$ $\quad \frac{66}{8}$ $\quad 71 \div 8$

$9\overline{)\,58}$ $\quad \frac{64}{9}$ $\quad 72 \div 9$

$2\overline{)\,17}$ $\quad \frac{19}{2}$ $\quad 10 \div 3$

$4\overline{)\,14}$ $\quad \frac{17}{4}$ $\quad 20 \div 4$

$5\overline{)\,15}$ $\quad \frac{18}{5}$ $\quad 21 \div 5$

$5\overline{)\,42}$ $\quad \frac{45}{5}$ $\quad 13 \div 6$

$6\overline{)\,41}$ $\quad \frac{45}{6}$ $\quad 49 \div 6$

$7\overline{)\,35}$ $\quad \frac{41}{7}$ $\quad 46 \div 7$

$8\overline{)\,27}$ $\quad \frac{33}{8}$ $\quad 38 \div 8$

$9\overline{)\,15}$ $\quad \frac{22}{9}$ $\quad 30 \div 9$

$9\overline{)\,78}$ $\quad \frac{81}{9}$

$3\overline{)\,14}$	$\frac{17}{3}$	$20 \div 3$
$4\overline{)\,23}$	$\frac{26}{4}$	$29 \div 4$
$5\overline{)\,24}$	$\frac{27}{5}$	$30 \div 5$
$6\overline{)\,17}$	$\frac{21}{6}$	$25 \div 6$
$6\overline{)\,52}$	$\frac{10}{7}$	$14 \div 7$
$7\overline{)\,52}$	$\frac{58}{7}$	$63 \div 7$
$8\overline{)\,44}$	$\frac{49}{8}$	$55 \div 8$
$9\overline{)\,37}$	$\frac{45}{9}$	$52 \div 9$

Problem: $170 \div 12 \qquad 12\overline{)170} \qquad \frac{170}{12}$

How many twelves are there in 170?

Clear your Numberaid. Put 170 (the dividend) on your Numberaid (Figure 105, Part A). You know that if you subtract 12 ten times from the dividend (170), you would have to subtract 120. You also can see that if you subtract 12 twenty times from the dividend, you would have to subtract 240. You know that the quotient is a number greater than 10 and less than 20. Subtract 120 from 170 (Figure 105, Part A). The remainder is 50 (Figure 105, Part B). To keep from forgetting the quotient, record it by a blue counter against the stop.

You know that if you subtracted 12 four times from the remainder, you would subtract 48. You could not subtract 12 five times for you would then be subtracting a number greater than the remainder. Subtracting 12 three times would leave a remainder greater than twelve, and would also be unsatisfactory. Subtract 48 from the remainder 50 on your Numberaid (Figure 105, Part B). Record the four of the quotient on your Numberaid by four yellow counters at the stop. The quotient is 14, the remainder is 2 (Figure 105, Part C).

$$170 \div 12 = 10 + 4\text{-}2/12 \text{ or } 14\text{-}2/12 \text{ or } 14\text{-}1/6$$

$$\begin{array}{r} 14\ \text{R}2 \\ 12\overline{)170} \end{array}$$

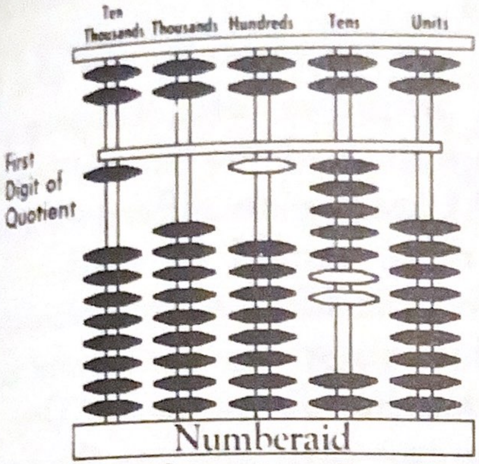
$$\frac{170}{12} = 14\text{-}2/12 \text{ or } 14\text{-}1/6$$

Now do the following problems:

$23\overline{)356} \qquad 18\overline{)607} \qquad 37\overline{)968} \qquad 15\overline{)225} \qquad 61\overline{)745}$

Figure 105 170 ÷ 12 = 14$\frac{2}{12}$

Part A



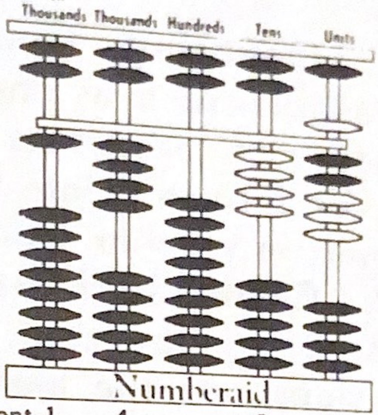
10 1 7 0

12)170 -1 2 0

120

Figure 105 170 ÷ 12 = 14$\frac{2}{12}$

Part B



Quotient 1 4 5 0

-4 8

14

12)170

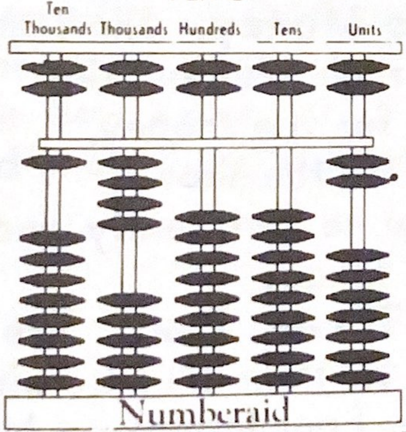
120

50

48

Figure 105 170 ÷ 12 = 14$\frac{2}{12}$

Part C



Quotient 1 4 Remainder 2

14 Quotient

12)170

120

50

48

2 Remainder

NUMBERAID GAMES

COUNTING

Game 1. Counting. Count various similar objects, one at a time, which are available, such as chairs, stairs, windows, etc., keeping a record on the paper of the names of the objects and the number counted. In each case, be sure to return to zero. At least nine difference kinds of objects, ranging in number from one to nine, should be used.

Game 2. Count to Miss. (Clapping Hands). All the Numberaids must be cleared to show zero. The leader claps his hands and counts aloud as he claps. As the leader claps his hands he says, "One, two, three," etc., at the best speed at which he can clap and say the numbers at the same time. The children keep count on their Numberaids. If they lose track of the count on their Numberaids, they drop out. The ones with the correct count when the leader stops clapping get one point. Five points make a game. If the leader makes a mistake, he is replaced by the child who notices the mistake and has the correct count on his Numberaid. The one who makes five points first becomes the leader. The Numberaid must be cleared before bginning again.

Game 3. One for the Money. The leader reads the rhyme. All the children keep count on their Numberaids.

One for the money,
Two for the show,
Three to get ready and
Four to go.

Game 4. One, Two, Three, Four. The leader reads the rhyme. All the children keep count on their Numberaids.

One, Two, Three, Four (count on Numberaid)
Mama scrubbed the kitchen floor (back to zero)
Floor dried, baby cried,
One, Two, Three, Four (count on Numberaid)

Game 5. Five Little Chickadees. The leader says the verses. All the children join in the chorus. Everyone keeps count on his Numberaid.

Five Little Chickadees,
Peeping at the door;
One flew away,
And then there were four.

Chorus: Chickadee, chickadee.
Happy and gay;
Chickadee, chickadee,
Fly away.

Four little chickadees,
Sitting on a tree;
One flew away
And then there were three.

Chorus

Three little chickadees,
Looking at you;
One flew away,
And then there were two.

Chorus

Two little chickadees,
Sitting in the sun;
One flew away,
And then there was one.

Chorus

One little chickadee,
Left all alone;
It flew away,
And then there were none.

Game 6. What is One? The leader asks the questions of different children. The one called upon tells something different to represent the number each time. All the children count to the number on their Numberaids, as the child who has been called on counts. Numberaids must be cleared between each verse.

"What is one?" askes the leader, pointing to Peter.
"That big pine tree against the sky, that is one," says Peter.
All clear Numberaids.
"What is two?" asks the leader, pointing to a second child.
"See our tracks behind us, two long tracks in the snow?
One, two. That is two," says Paul.
All clear Numberaids.
"What is three?" the leader points to a third child.
"Here's a birthday cake with candles.
One, two, three, Linda is three!" says Sue.
All clear Numberaids.
"What is four?" the leader points to a fourth child.
"Count the boots, all in a row.
One, two, three, four. That is four," says John.
All clear Numberaids.
"What is five?" asks the leader, pointing to a fifth child.
"A brown cat, a yellow cat, a mother cat, and two black kittens.
One, two, three, four, five. That is five," says Charles.
All clear Numberaids.
"But Ann, what is six?" asks the leader, pointing to a sixth child.
"See the goose, and father gander, and four grey goslings coming along behind? One, two, three, four, five, six. That is six," says Ann.
All clear Numberaids.

"What is seven?" the leader asks, pointing to a seventh child.

"Here's a nest with seven eggs. One, two, three, four, five, six, seven. That is seven," says Mark.

All clear Numberaids.

"What is eight?" says the leader, pointing to another child.

"Count the little goats dancing in the yard. One, two, three, four, five, six, seven, eight. That is eight," says Sharon.

All clear Numberaids.

"What is nine?" the leader asks, pointing to a ninth child.

"There are nine fine pigs in the pigpen there. One, two, three, four, five, six, seven, eight, nine. That is nine," says Bob. "Let's start again."

All clear Numberaids.

The game can be played over and over.

Game 7. Tennessee. The leader reads the rhyme. All keep count to nine on their Numberaids. The last line is either read "Tennessee" or "Ten I see." The children who respond incorrctly to the last line lose the game.

One I see
Two I see
Three I see
Four I see
Five I see
Six I see
Seven I see
Eight I see
Nine I see
Tennessee, or
Ten I see

Game 8. Ten Little Indians. Tell the children to imagine a row of little Indians. Sing or say the following rhymes as they keep count on their Numberaids. Be sure the Numberaids are cleared before the game is started.

The first little,
The second little,
The third little Indian;
The fourth little,
The fifth little,
The sixth little Indian;
The seventh little,
The eighth little,
The ninth little Indian;
The tenth little Indian Boy.

Ten little Indians, standing in a line,
The tenth went home, that left nine;
Nine little Indians, swinging on a gate,
The ninth fell off, then there were eight;
Eight little Indians, never heard of heaven,
The eighth kicked the bucket, then there were seven;
Seven little Indians, cutting up tricks,
The seventh broke his neck, then there were six;
Six little Indians, kicking all alive,
The sixth went to bed and that left five;
Five little Indians, on a cellar door,
The fifth tumbled in, and that left four;
Four little Indians out on a spree,
The fourth got sick, and that left three;
Three little Indians in a big canoe,
The third tumbled overboard and that left two;

Two little Indians, fooled with a gun,
The second shot the first and that left one;
One little Indian, the very last one,
Died of sorrow and now there are none.

Game 9. Barney. The leader reads the rhyme. One of the children plays Barney. This child pantomimes the rhyme, while the other children keep count on their Numberaids. The child keeps count until a pantomime is missed or the rhyme is ended. If a pantomime is missed, the child playing Barney is replaced by one of the children who have the correct number of the rhyme on their Numberaids. Each child who completes the pantomime without error chooses another child to take his place. Each child who has the correct number at the end of a rhyme makes one point. Ten points make a game. The object of the game is speed and accuracy in counting.

When Barney was one
He sucked his thumb.
When Barney was two
He touched his shoe.
When Barney was three
He touched his knee.
When Barney was four
He touched the floor.
When Barney was five
He touched his side.
When Barney was six
He picked up sticks.
When Barney was seven
He looked at heaven.
When Barney was eight
He shut the gate.
When Barney was nine
He wanted a dime.
When Barney was ten
He chased the hen.
When Barney was eleven
He dreamed of heaven.
When Barney was twelve
He could serve himself.

Game 10. One, Two, Buckle My Shoe. The children keep count on their Numberaids as the rhyme progresses. The leader recites the rhyme. As a number is missed, the player drops out. Those who are in the game at the end of the rhyme, and have the correct number, are given one point each. Five points make a game. The one saying the rhyme can stop at any number he wishes. If the leader makes a mistake, he is replaced by the one who notices it and has the correct number on his Numberaid.

One, two,
Buckle my shoe;
Three, four,
Shut the door;
Five, six,
Pick up sticks;
Seven, eight,
Lay them straight;
Nine, ten,
Big fat hen;
Eleven, twelve,
Dig and delve;
Thirteen, fourteen,
Maids are courting;
Fifteen, sixteen,
Maids are kissing;
Seventeen, eighteen,
Maids are waiting;
Nineteen, twenty,
That's a plenty;
Twenty-one, twenty-two,
That will do.

Game 11. Numbers. Choose a leader. The leader says, "Clear the Numberaid," and then calls out a number from one to nine. The first player to raise his hand, and who has the correct quantity on the Numberaid, scores one point. If it is wrong, he loses one point. The first player to score ten points takes the leader's place. Then the game is begun. Three or more players are needed for the game.

Game 12. Count to Miss (Bouncing Ball). The leader bounces a ball three times so that the children can see and hear the group of three. The children put the number three on their Numberaids after the leader has finished. The game is intended to develop skill in recognizing groups. Those who have the wrong number for a group, drop out of the game. The leader then bounces the ball another number of times. The children must wait until the leader has finished bouncing it before putting the number on their Numberaids. One point is given for each correct answer. The one who gets ten points takes the leader's place. If he makes a mistake, the child who discovered the mistake takes over the leader's job. An extension of this game can be played, in which the leader bounces the ball in equal groups and the children count the number of groups. This can lead to an understanding of the multiplication process. When the leader stops, the children who have recorded the correct number of groups on their Numberaids win a point. The game is ten points, and the winner takes the leader's place.

Game 13. Imagination. The leader says, "Imagine that I have one apple and another and another. Write the number which shows how many apples I have on your Numberaid. The Numberaids are inspected. The children who did not show the right number lose by a point; those who did, gain one. The leader continues counting imaginary objects of various kinds, and the children write the total number on their Numberaids. Ten points make a game. The winner takes the leader's place.

Game 14. One Little Girl Has One Little Cat. The leader reads the rhymes. All the children put the number on their Numberaids at the beginning of each verse. The Numberaids should be **cleared** at the end of each verse. The numbers should be put on the Numberaids in groups, not as individual counters. "Two" means two counters at a time, not one at a time. The leader can stop at any number for "inspection." Those who have numbers wrong are out of the game. The ones who get all correct win a point. Five points is a game. The winner becomes the leader.

Clear the Numberaid.
One little girl has one little cat,
A new pink dress and a big straw hat!
Clear the Numberaid.
Two little brothers looking just alike,
One's name is Bill, and one's name is Mike!
Clear the Numberaid.
Three little kittens wear their mittens,
When they go out to play.
Three little kittens wear their mittens,
Every single day.
Clear the Numberaid.
Four frisky puppy dogs
Playing in the sun.
Two like to roll
And two like to run.
Clear the Numberaid.
Five honeybees are buzzing
Round the garden hive;
Sally Ann can count them,
One, two, three, four, five!

Clear the Numberaid.
Six cheery little chipmunks
Doing funny tricks;
One by one you count them . . .
That is right, just six.
Seven
Oh, my! Seven o'clock, and
Seven sleepyheads . . .
Ted, May, Bob, Dick, Jane, Sue, Ann . . .
Hurry to their beds.
Eight pretty bluebirds
Live in an apple tree;
Twitter, twitter, twitter,
They sing a song to me.
Nine
Out in the garden
When the day is sunny
Nine yellow butterflies,
Come to sip honey.

ADDITION

Game 15. Post Office. The leader says, "I have to buy some stamps. I need four one-cent stamps and two two-cent stamps. How much must I pay the postmaster?" The first child with the correct answer gets one point. Five points entitles any child to become the leader. The game is played within the skill of the children in addition.

Game 16. Grocery Store. The leader says, "I am going to the grocery store. I need one penny for gum, two pennies for a piece of candy, and a nickel for an ice cream cone. How much money must I ask Mother for? The child who gives the correct answer first wins a point. Five points is a game. The winner of the game takes the place of the leader.

Game 17. Spinner. A spinner (a card with a pointer which spins on a pin) with the numbers 0 1 2 3 4 5 6 7 8 9 is needed to play this game. Each child is given a number from one to nine by chance. A goal number such as 9 is set. The spinner is passed from child to child. As each child spins he tells the group what the number the pointer stopped at was. This number is added to the number he originally was given. The first child to get to the goal number wins. He must reach it exactly. Ten points make a game. The numbers given and the goal should be kept within the skill and understanding of the children.

Game 18. Add a Number. The leader calls out a number. He then calls out a second number. The child who gets the correct sum first wins the point. Ten points makes a game. The one who wins takes the leader's place.

MULTIPLICATION

Game 19. Count to Miss (Bouncing Ball). The leader bounces a ball so that the children can see and hear it. He counts aloud by twos, as he bounces the ball. The children keep count on their Numberaids. If the leader makes a mistake, a child who has the correct total on his Numberaid hakes his place. The complexity of the game should be kept winthin the skill and understanding of the children. This game is intended to develop skill in counting by groups, and will help children to remember their multiplication facts. They can be varied by using numbers from 1 to 9 and later from 10 to 90 and from 100 to 900, using one bounce to represent ten or one hundred as desired.

Game 20. Speedy. The leader tells the players to multiply a number, such as two, a specified number of times. The first player to get the correct answer on his Numberaid scores one point. The correct answer is given by the leader, who must do the calculation mentally. If the leader is wrong, a player who has the correct answer takes his place. The player who scores ten wins the round and becomes the leader. Different numbers should be used by the leader for each problem. The complexity of the problems should be kept within the skill and understanding of the children.

SUBTRACTION

Game 21. Post Office. The leader says, "I have to buy some stamps. I have twenty cents. Give me three one-cent stamps, four two-cent stamps and one three-cent stamp. How much change will I get?" The first child with the correct answer on his Numberaid wins. Ten points make a game. The numbers involved can be chosen to meet the skill and understanding of the children.

Game 22. Grocery Store. The leader says, "I am going to the grocery store. I have fifty cents. I will buy a ten-cent candy bar, a ruler for ten cents, a new pencil for five cents, and a pad of paper for twelve cents. How much money should I have left?"

Game 23. Subtraction. The spinner card is used again. The game is the reverse of addition. A fixed number is used by all players as a starting point. Each number obtained by each player as he spins the arrow is subtracted from the fixed number. When zero is finally attained by a player, he wins the game. The size of the fixed number with which the game is begun should be within the skill and understanding of the children.

Game 24. The Missing Number. The leader calls out a larger number and a smaller number. The child who first puts on his Numberaid the number which is the difference between the larger and the smaller numbers wins the point. The first to get ten points becomes the leader.

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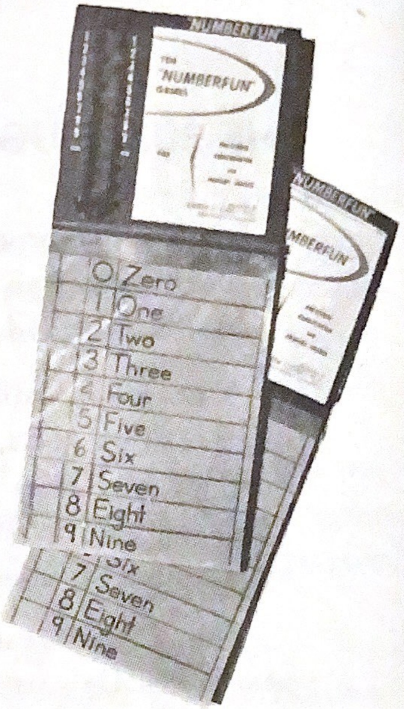

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